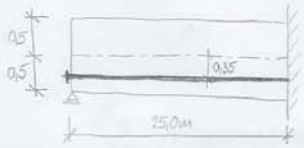
①



$P = P_H = 1000\,kN$

SN 1x

$x = \overline{M}_b$

$$\delta_{11} = \varphi_{0a} + \overline{M}_b \cdot \alpha_{ba} + M_a \cdot \beta_{ab} + M_b \cdot \alpha_{aa} = 0$$

$$\overline{M}_b \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{6EJ} - 350 \cdot \frac{25}{3EJ} = 0$$

$$\overline{M}_b = 525\,kNm$$

$\alpha = \frac{L}{3EJ}$ $\quad \beta = \frac{L}{6EJ}$

$$R_a \cdot 25 = \overline{M}_b \Rightarrow R_a = 21\,kN$$

$$R_b = -21\,kN$$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

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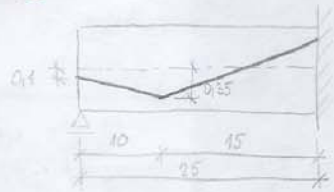
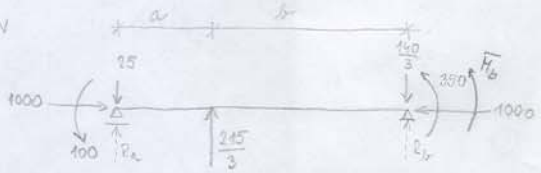
②



$P = P_H = 1000\,kN$

SN 1x

$x = \overline{M}_b$



$$\frac{0,25}{10} = \frac{P_{V1}}{1000} \Rightarrow P_{V1} = 25\,kN$$

$$\frac{0,70}{15} = \frac{P_{V2}}{1000} \Rightarrow P_{V2} = \frac{140}{3}\,kN$$

$$P_V = P_{V1} + P_{V2} = \frac{215}{3}$$

$$\varphi_{ba} + M_a \cdot \beta_{ab} + M_b \cdot \alpha_{ab} + \overline{M}_b \cdot \alpha_{ab} = 0$$

$$-\frac{75 \cdot 25}{3EJ} - 100 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{M}_b \cdot \frac{25}{3EJ} = 0$$

$$\overline{M}_b = 1\,kNm$$

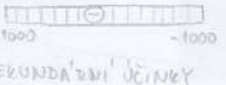
$$25 \cdot R_a - 100 \cdot 25 - 25 \cdot 25 + \frac{215}{3} \cdot 15 - 350 - 1 = 0$$

$$R_a = 0,04\,kN \qquad R_b = -0,04\,kN$$

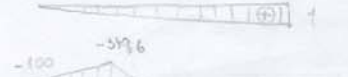
$$\varphi_{0a} = \frac{F \cdot a_1}{6EJ}\left(L^2 - a_1^2\right) =$$

$$= -\frac{215}{3} \cdot \frac{10}{6EJ \cdot 25} \cdot \left(25^2 - 10^2\right) = -\frac{7525}{3EJ}$$

PRIMÁRNÍ ÚČINKY

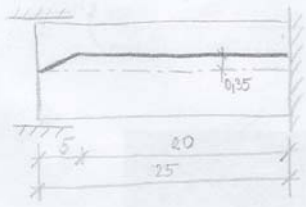


SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY

③



$P = P_H = 1000kN$

SN 2x

$X_1 = \overline{H}_a \quad X_2 = \overline{H}_b$

$\frac{0,35}{5} = \frac{P_V}{100}$ $\quad P_V = 70kN$

$\varphi_{aa} = \frac{F \cdot a}{6EJL}(L^2 - a^2) = \frac{20 \cdot 20}{6EJ \cdot 25}(25^2 - 20^2) = \frac{2100}{EJ}$

$\varphi_{ba} = \frac{F \cdot a}{6EJL}(L^2 - a^2) = \frac{20 \cdot 5}{6EJ \cdot 25}(25^2 - 5^2) = \frac{1400}{EJ}$

(1) $\varphi_{aa} + \overline{H}_a \cdot \alpha_{aa} + \overline{H}_b \cdot \beta_{ba} + H_A \cdot \beta_{aa} = 0$

$\frac{2100}{EJ} + \overline{H}_a \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{6EJ} = 0$

(2) $\varphi_{ba} + \overline{H}_a \cdot \beta_{aa} + \overline{H}_b \cdot \alpha_{ba} + H_B \cdot \alpha_{ba} = 0$

$\frac{1400}{EJ} + \overline{H}_a \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{3EJ} = 0$ $\quad \cdot(-\frac{1}{2})$

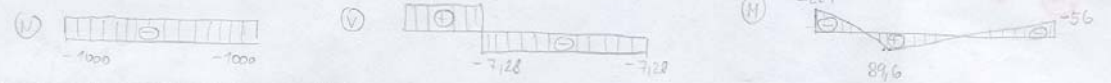
$\frac{350}{EJ} + 0 + \overline{H}_b \cdot \frac{25}{4EJ} + \frac{4375}{2EJ} = 0 \Rightarrow \overline{H}_b = -406kNm \quad \overline{H}_a = -224kNm$

$25 \cdot R_a + 70 \cdot 25 - 224 - 70 \cdot 20 - 350 + 406 = 0 \quad \ldots \quad R_a = -7,28kN \quad R_b = 7,28kN$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



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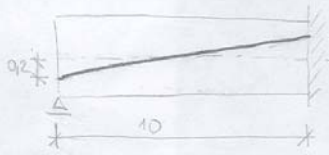
④



$P = P_H = 1000kN$

SN 1x

$X = \overline{H}_b$

$0 = \varphi_{ba} + \overline{H}_b \cdot \alpha_{ba} + H_B \cdot \alpha_{ba} + H_a \cdot \beta_{aa} = 0$

$\overline{H}_b \cdot \frac{10}{3EJ} + 200 \cdot \frac{10}{3EJ} - 200 \cdot \frac{10}{6EJ} = 0 \Rightarrow \overline{H}_b = -100kN$

$R_a \cdot 10 = \overline{H}_b = 100 \rightarrow R_a = 10kN \ (\downarrow) \quad R_b = 10kN \ (\uparrow)$

$\frac{P_V}{1000} = \frac{0,2}{5} \quad P_V = 40kN$

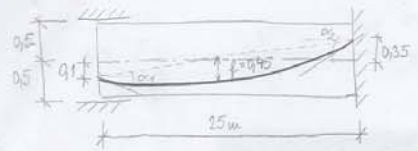
PRIMÁRNÍ ÚČINKY

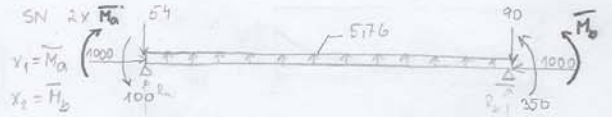
SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

(12)

$P = P_H = 1000\,kN$





$$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(e_2 - e_1)}{l}\cdot x$$

$$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{(e_2 - e_1)}{l}$$

$$y'(0) = \operatorname{tg}\alpha_1 = \frac{4f}{l} + \frac{(e_2 - e_1)}{l} = \frac{4\cdot 0{,}45}{25} + \frac{(-0{,}35 - 0{,}1)}{25} =$$

$$= 0{,}054 \ldots\ldots P_{v1} = 54\,kN$$

$$y'(25) = \operatorname{tg}\alpha_2 = -\frac{8\cdot 0{,}45\cdot 25}{25^2} + \frac{4\cdot 0{,}45}{25} + \frac{(-0{,}35 - 0{,}1)}{25} =$$

$$= -0{,}09 \ldots\ldots P_{v2} = 90\,kN$$

$$p = \frac{8\cdot P_H\cdot f}{l^2} = \frac{8\cdot 1000\cdot 0{,}45}{25^2} = 5{,}76\,kNm^{-1}$$

$$\varphi_{ab} = \varphi_{ba} = \frac{1}{24}\frac{pl^3}{EJ} = \frac{1}{24}\cdot\frac{(-5{,}76)\cdot 25^3}{EJ} = -\frac{3750}{EJ}$$

(1) $\varphi_{ab} + \overline{H_a}\cdot\alpha_{ab} + H_a\,\alpha_{ab} + H_b\cdot\beta_{ba} + \overline{H_b}\cdot\beta_{ba} = 0$

$$-\frac{3750}{EJ} + \overline{H_a}\cdot\frac{25}{3EJ} - 100\cdot\frac{25}{3EJ} + 350\cdot\frac{25}{6EJ} + \overline{H_b}\cdot\frac{25}{6EJ} = 0$$

(2) $\varphi_{ba} + \overline{H_a}\cdot\beta_{ab} + H_a\cdot\beta_{ab} + H_b\cdot\alpha_{ba} + \overline{H_b}\cdot\alpha_{ba} = 0$

$$-\frac{3750}{EJ} + \overline{H_a}\cdot\frac{25}{6EJ} - 100\cdot\frac{25}{6EJ} + 350\cdot\frac{25}{3EJ} + \overline{H_b}\cdot\frac{25}{3EJ} = 0$$ $\quad (-\tfrac{1}{2})$

$$-\frac{1875}{EJ} + 0 + 0 + \frac{1375}{2EJ} + \overline{H_b}\cdot\frac{25}{4EJ} = 0$$

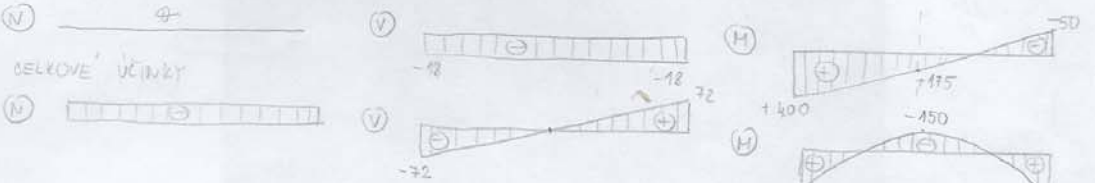
$\overline{H_b} = -50\,kNm$ $\qquad$ $\overline{H_a} = 400\,kNm$

$100 + 25\cdot R_a + 50 = 0 \Rightarrow$ $R_a = 18\,kN\ (\downarrow)$

$25\cdot R_b - 50 - 100 = 0$ $\qquad$ $R_b = +18\,kN\ (\uparrow)$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY



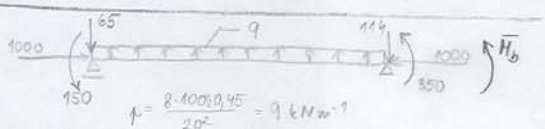
CELKOVÉ ÚČINKY

(13)

$f = 0{,}45$

$P = P_H = 1000\,kN$

SN 1x



$$p = \frac{8\cdot 1000\cdot 0{,}45}{20^2} = 9\,kNm^{-1}$$

$$y'(0) = \operatorname{tg}\alpha_1 = \frac{4\cdot 0{,}45}{20} + \frac{(-0{,}35 - 0{,}15)}{20} = 0{,}065 \ldots P_{v1} = 65\,kN$$

$$y'(20) = \operatorname{tg}\alpha_2 = -\frac{8\cdot 0{,}45\cdot 20}{20^2} + \frac{4\cdot 0{,}45}{20} - \frac{0{,}5}{20} = -0{,}115 \ldots P_{v2} = 114\,kN$$

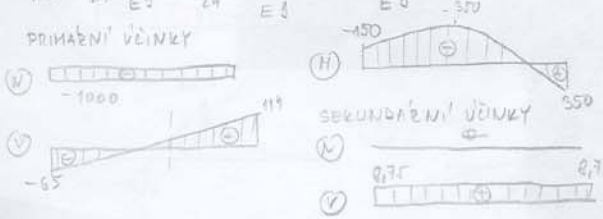
$$\varphi_{ba} = \frac{1}{24}\frac{pl^3}{EJ} = \frac{1}{24}\cdot\frac{(-9)\cdot 20^3}{EJ} = -\frac{3000}{EJ}$$

$$\varphi_{ba} + H_a\cdot\beta_{ab} + H_b\cdot\alpha_{ba} + \overline{H_b}\cdot\alpha_{ba} = 0$$

$$-\frac{3000}{EJ} - 150\cdot\frac{20}{6EJ} + 350\cdot\frac{20}{3EJ} + \overline{H_b}\cdot\frac{20}{3EJ} = 0$$

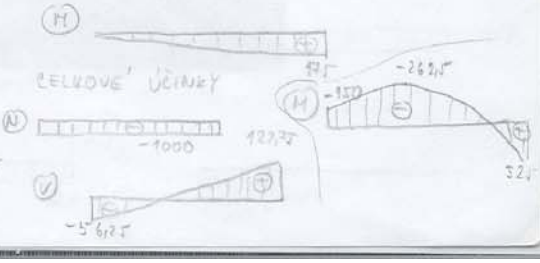
$\overline{H_b} = 175\,kNm$ $\qquad$ $R_a = 8{,}75\,kN\ (\uparrow)$ $\qquad$ $R_b = +8{,}75\,kN\ (\downarrow)$

PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY

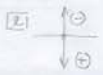
CELKOVÉ ÚČINKY



(10)

$P = 1000\,kN$

SN 0 X



$$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(l_2 - l_1)}{l}\cdot x$$

$$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{(l_2 - l_1)}{l}\cdot x$$

$$y(0) = tg\,\alpha_1 = tg\,\alpha_2 = \frac{4f}{l} = \frac{4\cdot 0{,}8}{20} = 0{,}16$$

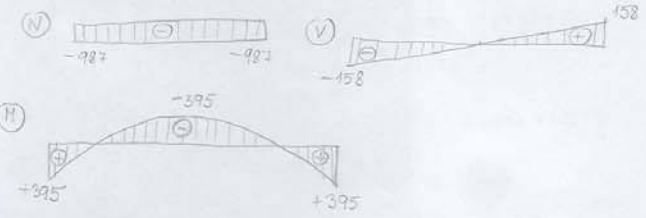
$P_V = P\cdot \sin\alpha = 158\,kN$

$P_H = P\cdot \cos\alpha = 987\,kN$

$$p = \frac{8\cdot P_H\cdot f}{l^2} = \frac{8\cdot 987\cdot 0{,}8}{20^2} = 15{,}8\,kN\,m^{-1}$$

$H = 0{,}4\cdot P_H = 395\,kNm$

PRIMÁRNÍ ÚČINKY (= CELKOVÉ ÚČINKY)



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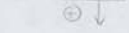
(11) (8)

$P = 1000\,kN$

SN 1x

$x = \overline{H_b}$



$$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(l_2 - l_1)}{l}\cdot x$$

$$tg\,\alpha = \frac{2\cdot 0{,}85}{10} = 0{,}17$$

$P_V = P\cdot \sin\alpha = 167{,}6\,kN$

$P_H = P = 1000\,kN$

$$p = \frac{8\cdot P_H\cdot f}{l^2} = \frac{8\cdot 1000\cdot 0{,}25}{20^2} = 17\,kN\,m^{-1}$$

$H_a = P_H\cdot 0{,}5 = 500\,kNm$

φ_{ba}:

$$\varphi^1_{ba} = \frac{q a^2 l}{6EJ}\left(1 - \frac{a}{2l}\right)^2 = \frac{17\cdot 10^2\cdot 25}{6EJ}\left(1 - \frac{10}{2\cdot 25}\right)^2 = \frac{13600}{3EJ}$$

$$\varphi^2_{ba} = \frac{F\cdot a}{6EJl}\left(l^2 - a^2\right) = -\frac{167{,}6\cdot 15}{6EJ\cdot 25}\left(25^2 - 15^2\right) = -\frac{6704}{EJ}$$

$$\varphi^3_{ba} = \frac{Ml}{6EJ}\left(1 - 3\cdot\frac{a^2}{l^2}\right) = -\frac{500\cdot 25}{6EJ}\left(1 - 3\cdot\frac{15^2}{25^2}\right) = +\frac{500}{3EJ}$$

$$\varphi_{ba} = -\frac{6012}{3EJ}$$

$$\varphi_{ba} + \overline{H_b}\cdot\alpha_{ba} + \overline{H_b}\cdot\alpha_{bb} = 0$$

$$-\frac{6012}{3EJ} + \overline{H_b}\,\frac{25}{3EJ} + 350\cdot\frac{25}{2EJ} = 0$$

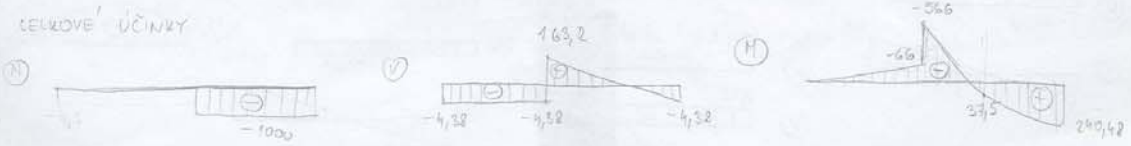
$\overline{H_b} = -109{,}52\,kNm$

$R_a = 4{,}4\,kN\ (\downarrow)$ $\quad R_b = 4{,}4\,kN\ (\uparrow)$

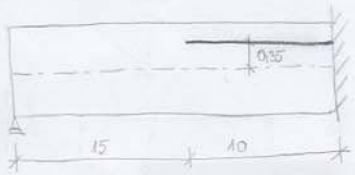
PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



(5) (11)



$P = P_H = 1000\,kN$

SN 1x

$X = \overline{H}_b$

$$\varphi_{ba} = \frac{M \cdot L}{GEJ}\left(1 - 3\frac{a^2}{L^2}\right) = \frac{350 \cdot 25}{GEJ}\left(1 - 3 \cdot \frac{15^2}{25^2}\right) = -\frac{350}{3EJ}$$

$$\varphi_{ba} + \overline{H}_b \cdot \alpha_{bb} + H_A \cdot \alpha_{ba} = 0$$

$$-\frac{350}{3EJ} + \overline{H}_b \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{3EJ} = 0 \Rightarrow \underline{\overline{H}_b = -336\,kNm}$$

$$25 \cdot R_a = 336 \Rightarrow \underline{R_a = 13{,}44\,kN\ (\downarrow)} \qquad \underline{R_b = 13{,}44\,kN\ (\uparrow)}$$

PRIMÁRNÍ ÚČINKY

(N)

(V)

(M)

SEKUNDÁRNÍ ÚČINKY

(N)

(V)

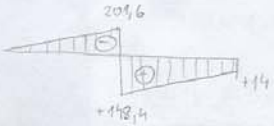
(M)

CELKOVÉ ÚČINKY

(N)

(V)

(M)



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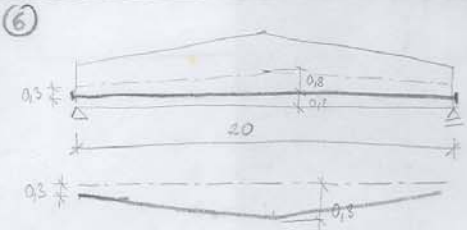
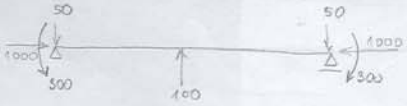
(6)



$P = P_H = 1000\,kN$

SN 0x



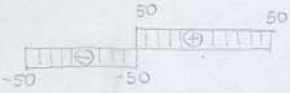
$$\frac{P_V}{1000} = \frac{0{,}5}{10}$$

$P_V = 50\,kN$

PRIMÁRNÍ ÚČINKY (= CELKOVÉ ÚČINKY)

(N)

(V)



(M)

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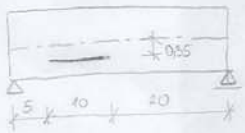
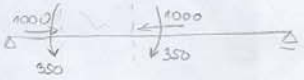
(7)



$P = P_H = 1000\,kN$

SN 0x



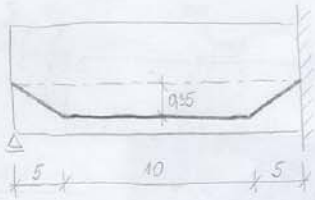
PRIMÁRNÍ ÚČINKY (= CELKOVÉ ÚČINKY)

(N)

(V)

(M)

⑧



$p = P_H = konst.$

SN: 1x

$X = \overline{H}_b$

$\frac{P_V}{1000} = \frac{0,35}{5}$ $\quad P_V = 70kN$

$\varphi_{1a} + \overline{H}_b \cdot \alpha_{1a} = 0$ $\quad \varphi_{1a} = \frac{3}{32} \cdot \frac{FL^2}{EJ} = \frac{3}{32} \cdot \frac{(-70)}{EJ} \cdot 20^2 =$

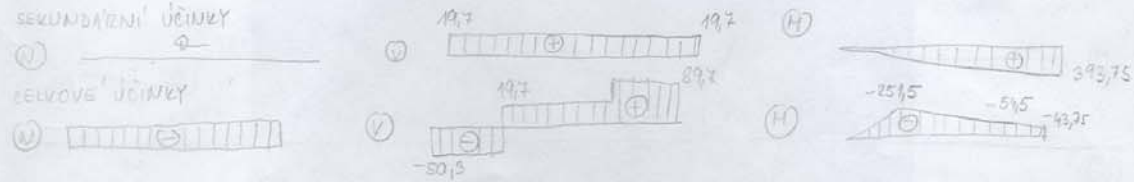
$-\frac{2625}{EJ} + \overline{H}_b \cdot \frac{20}{3EJ} = 0$ $\quad = -\frac{2625}{EJ}$

$\overline{H}_b = 393,75 kNm$

$20 R_a = 393,75$ ..... $R_a = 19,7 kN (\uparrow)$ $\quad R_b = 19,7 kN (\downarrow)$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY

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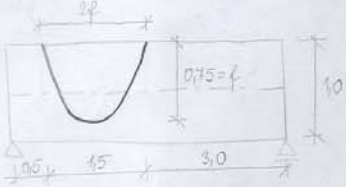
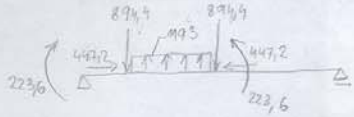
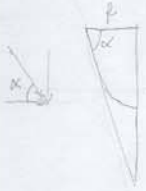
⑨



$p = 1000 kN$

SN: 0x





$\frac{2f}{f} = 2$

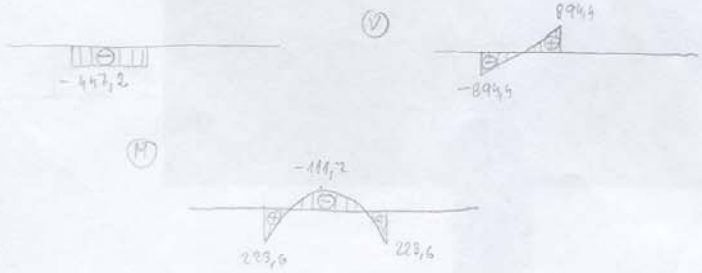
$P_V = P \cdot \sin\alpha = 894,4 kN$

$P_H = P \cdot \cos\alpha = 447,2 kN$

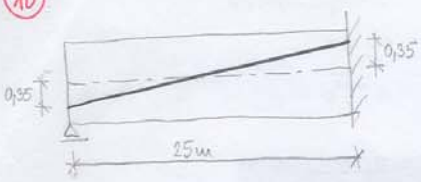
$p = \frac{8 \cdot P_H \cdot f}{L^2} = \frac{8 \cdot 447,2 \cdot 0,75}{1,5^2} = 1193 kNm^{-1}$

$H = P_H \cdot 0,5 = 223,6 kNm$

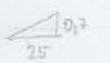
PRIMÁRNÍ ÚČINKY (= CELKOVÉ ÚČINKY)



(10)



$P = P_H = 1000\,kN$

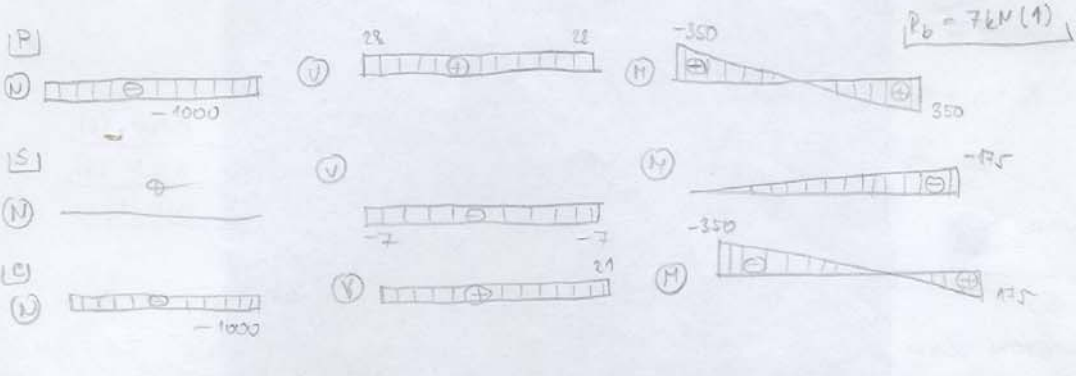


$P_V = 1000 \cdot \frac{0,7}{25} = 28\,kN$

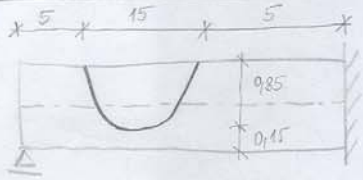
$-350 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \bar{H}_b \cdot \frac{25}{3EJ} = 0 \Rightarrow \bar{H}_b = -175\,kNm$

$25 \cdot R_a + 28 \cdot 25 - 350 - 350 + 175 = 0 \dots R_a = -7\,kN\,(\downarrow)$

$R_b = 7\,kN\,(\uparrow)$



(12)



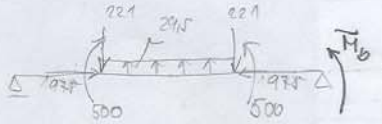
$tg\alpha = \frac{2 \cdot 0,85}{\frac{15}{2}}$

$P_V = P \cdot \sin\alpha = 221\,kN$

$P_H = P \cdot \cos\alpha = 975\,kN$

$q = \frac{8 \cdot 975 \cdot 0,85}{15^2} = 29,5\,kNm^{-1}$

1 x SN



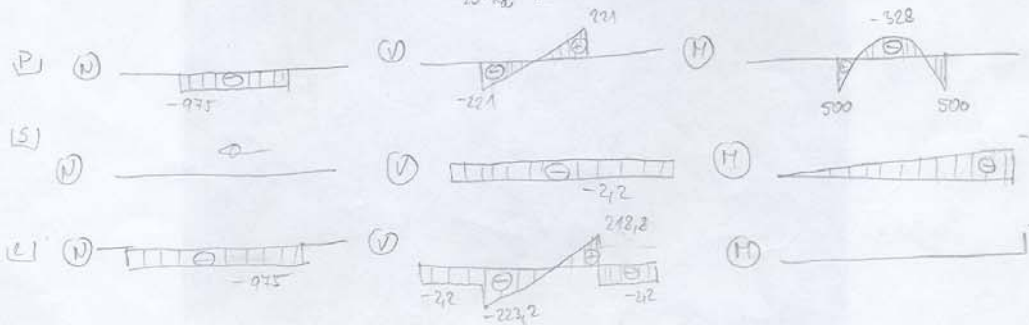
$\varphi_{ba} = -\frac{29,5}{24EJ} \cdot \frac{12,5 \cdot 15}{25} \left[4 \cdot 12,5\,(25 + 12,5) - 15^2\right] + \frac{221 \cdot 5}{2EJ}(25 - 5) +$

$+ 500 \cdot \frac{25}{6EJ}\left(1 - 3 \cdot \frac{5^2}{25^2}\right) - 500 \cdot \frac{25}{6EJ}\left(1 - 3 \cdot \frac{20^2}{25^2}\right) =$

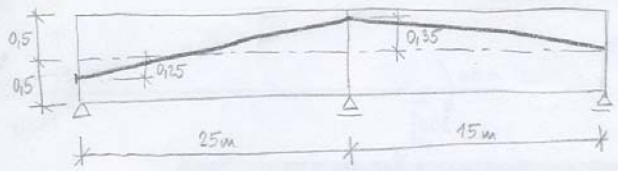
$= -\frac{410,9375}{EJ}$

$\bar{H}_b \frac{25}{3EJ} - \frac{410,9375}{EJ} = 0 \Rightarrow \bar{H}_b = -49\,kNm$

$25 \cdot R_a - 221 \cdot 20 + 29,5 \cdot 15 \cdot \left(\frac{15}{2} + 5\right) - 221 \cdot 5 + 49 = 0 \dots R_a = -2,2\,kN\,(\downarrow)$



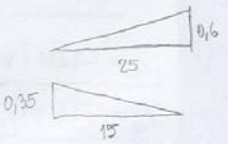
16



$P = P_H = 1000\,kN$

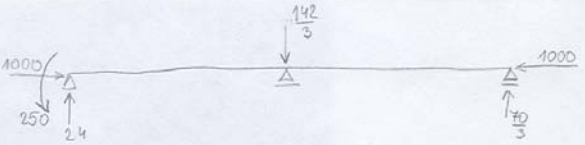
SN 1x

$X = \overline{M}_b$



$\frac{0,6}{25} = \frac{P_V}{1000}$ ..... $P_V = 24\,kN$

$\frac{0,35}{15} = \frac{P_V}{1000}$ ..... $P_V = \frac{70}{3}$



$\varphi_{ba} + \varphi_{bc} + H_b \cdot \beta_{bb} + \overline{M}_b (\alpha_{ba} + \alpha_{bc}) = 0$

$-250 \cdot \frac{25}{6EJ} + \overline{M}_b \left(\frac{25}{3EJ} + \frac{15}{3EJ} \right) = 0$

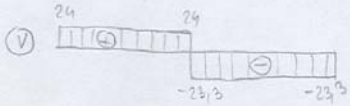
$\overline{M}_b = 78,1\,kNm$

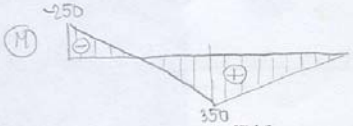
$25 \cdot R_a + 24 \cdot 25 - 250 - 78,1 = 0$ ..... $R_a = -10,9\,kN \ (\downarrow)$

$15 \cdot R_c + \frac{70}{3} \cdot 15 - 78,1 = 0$ ..... $R_c = -18,1\,kN \ (\downarrow)$

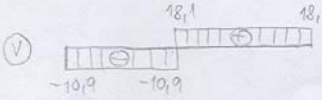
$R_A = 29\,kN \ (\uparrow)$

PRIMÁRNÍ ÚČINKY

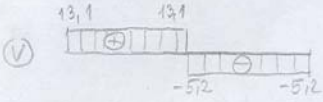


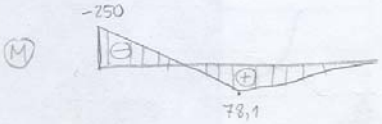


SEKUNDÁRNÍ ÚČINKY

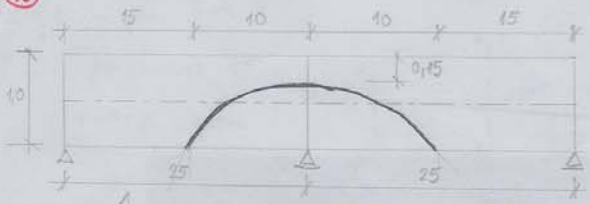


CELKOVÉ ÚČINKY





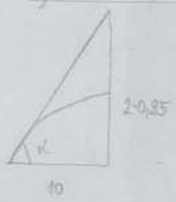
(15) (15)



$P = P_1 = 1000\ kN$

SN 1x ... $X = \overline{H}_b$

$f = 0{,}85$



$tg\alpha = \frac{2 \cdot 0{,}85}{10} = 0{,}17$

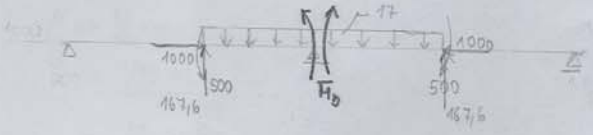
$P_V = P_{max} = 167{,}6\ kN$

$p = \frac{8 \cdot P_1 \cdot f}{l^2} = \frac{8 \cdot 1000 \cdot 0{,}85}{20^2} = 17\ kNm^{-1}$

q ... M

$\varphi_{ba} = \frac{17 \cdot 10^3 \cdot 25}{6EJ}\left(1 - \frac{10}{2 \cdot 25}\right)^2 - \frac{500 \cdot 25}{6EJ}\left(1 - 3 \cdot \frac{15^2}{25^2}\right) -$

$- \underbrace{\frac{167{,}6 \cdot 15}{6EJ \cdot 25}\left(25^2 - 15^2\right)}_{F} = -\frac{2004}{EJ}$



$\varphi_{bc} = \frac{17 \cdot 10^3 \cdot 25}{12EJ}\left(1 - \frac{10^2}{2 \cdot 25^2}\right) - \frac{500 \cdot 25}{6EJ}\left(1 - 3 \cdot \frac{15^2}{25^2}\right) -$

$- \frac{167{,}6 \cdot 15}{6EJ \cdot 25}\left(25^2 - 15^2\right) = -\frac{3279}{EJ}$

$\varphi_{ba} + \varphi_{bc} + \overline{H}_b \cdot (\alpha_{ba} + \alpha_{bc}) = 0$

$-\frac{2004}{EJ} - \frac{3279}{EJ} + \overline{H}_b\left(\frac{25}{3EJ} + \frac{25}{3EJ}\right) = 0$

$\overline{H}_b = 316{,}98\ kNm$

$25 \cdot R_a + 167{,}6 \cdot 10 - 500 - \frac{1}{2} \cdot 17 \cdot 10^2 - 316{,}98 = 0$

$R_a = -0{,}36\ kN\ (\downarrow)$

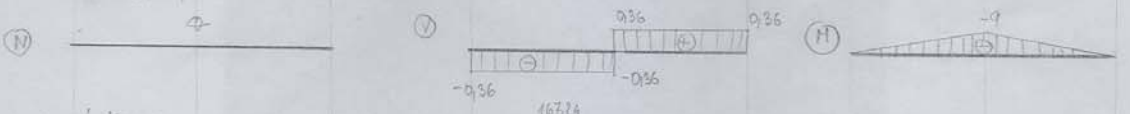
$25 \cdot R_c + 167{,}6 \cdot 10 - 500 - \frac{1}{2} \cdot 17 \cdot 10^2 - 316{,}98 = 0$

$R_c = -0{,}36\ kN\ (\downarrow)$

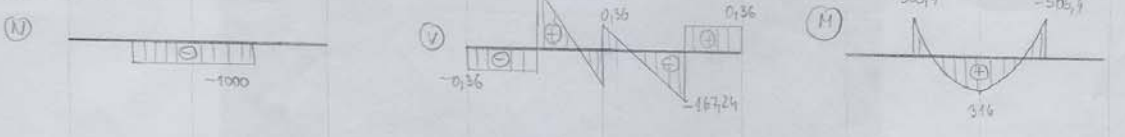
$R_b = 0{,}72\ kN\ (\uparrow)$

PRIMÁRNÍ ÚČINKY

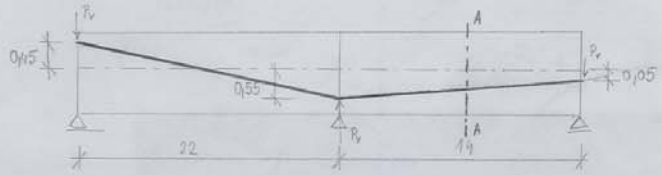
SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY



(16)



$P_H = P = ?$ aby x A-A $\sigma_\alpha = 0$

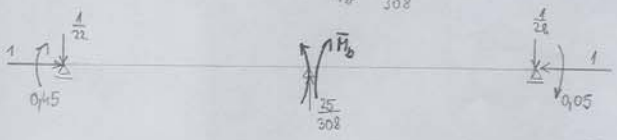
SV $1x \ldots X = \overline{H}_b$

volba: $P_H = P = 1$

$\frac{P_V}{1} = \frac{1}{22} \ldots P_V = \frac{1}{22}$

$\frac{P_V}{1} = \frac{0,5}{14} \ldots P_{V_C} = \frac{1}{28}$

$P_{Vb} = \frac{25}{308}$



$\frac{7C}{180} \cdot \frac{1}{14}$

$22 R_a - \frac{1}{22} \cdot 22 + 0,45 + \frac{23}{180} = 0$

$R_a = \frac{19}{990}$ kN (↑)

$M_a \cdot \beta_{ab} + \overline{H}_b (\alpha_{ba} + \alpha_{bc}) + M_c \cdot \beta_{cb} = 0$

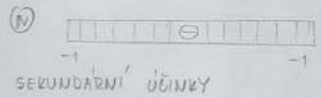
$0,45 \cdot \frac{22}{6EJ} + \overline{H}_b \left(\frac{22}{3EJ} + \frac{14}{3EJ} \right) - 0,05 \cdot \frac{14}{6EJ} = 0$

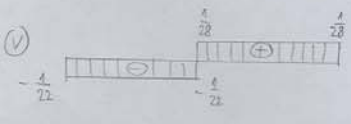
$\overline{H}_b = -\frac{23}{180}$ kN m

$14 \cdot R_b - 0,05 - \frac{1}{28} \cdot 14 + \frac{23}{180} = 0$

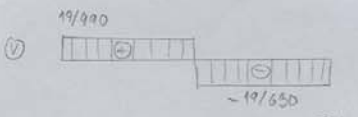
$R_b = \frac{19}{630}$ kN (↑)

PRIMÁRNÍ ÚČINKY

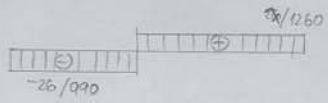


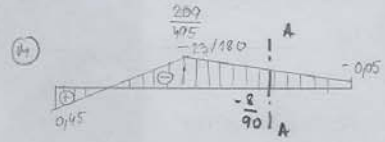


SEKUNDÁRNÍ ÚČINKY



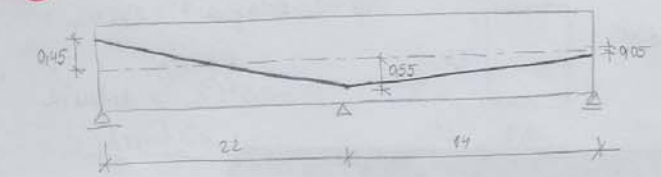
CELKOVÉ ÚČINKY





$\sigma = \frac{N}{A} + \frac{M}{W}$

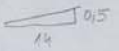
17



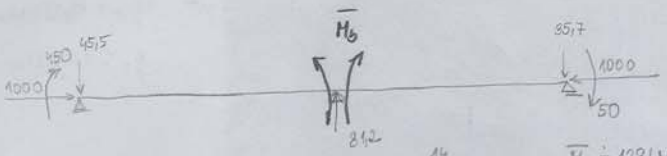
1 x SN

$P = P_H = 1000\,kN$

$P_{VA} = 1000 \cdot \frac{1}{22} = 45{,}45\,kN$



$P_{VC} = 1000 \cdot \frac{0{,}5}{14} = 35{,}7\,kN$

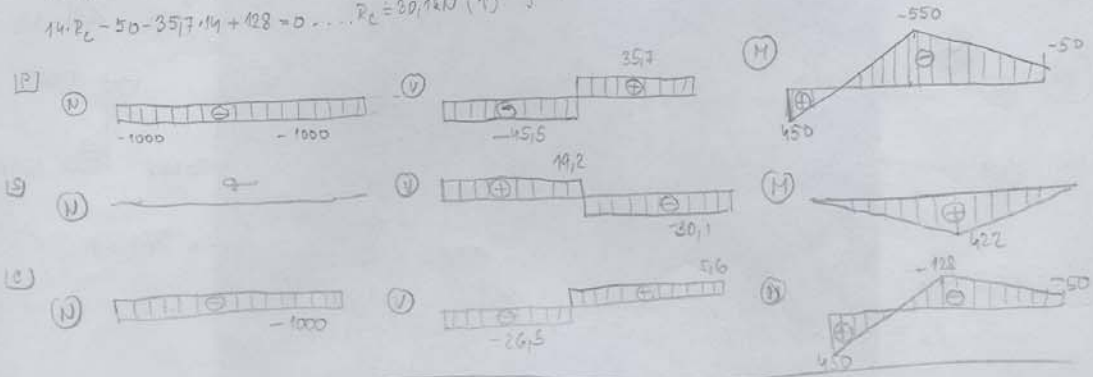


$$450 \cdot \frac{22}{6EJ} + \overline{H_b} \cdot \left(\frac{22}{3EJ} + \frac{14}{3EJ}\right) - 50 \cdot \frac{14}{6EJ} = 0 \quad \dots \quad \overline{H_b} = -128\,kNm$$

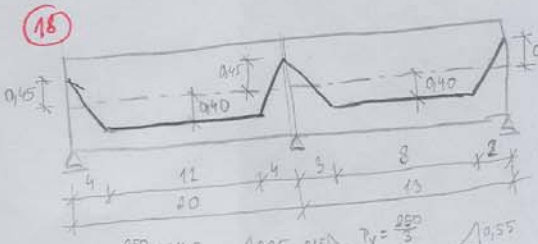
$22 \cdot R_a + 450 - 22 \cdot 45{,}5 + 128 = 0 \quad \dots \quad R_a = 19{,}2\,kN\ (\uparrow)$

$14 \cdot R_c - 50 - 35{,}7 \cdot 14 + 128 = 0 \quad \dots \quad R_c = 30{,}1\,kN\ (\uparrow)$

$\Big\} \; P_b = 49{,}3\,kN\ (\downarrow)$

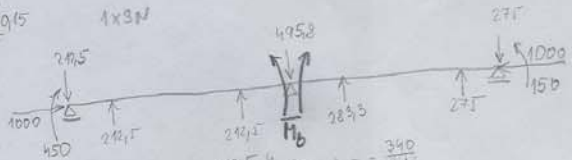


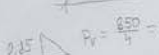
18



$P_H = P = 1000\,kN$

1 x SN





$$\varphi_{Ra} = \frac{F_a}{2EJ} \cdot (2 \cdot a) = -\frac{212{,}5 \cdot 4}{2 \cdot EJ \cdot 20}(20 - 4) = -\frac{340}{EJ}$$

$$\varphi_{Rc} = -\frac{283{,}3 \cdot 10}{6EJ \cdot 15}(15^2 - 10^2) - \frac{275 \cdot 2}{6EJ \cdot 15}(15^2 - 2^2) = -\frac{3670}{EJ}$$

$$450 \cdot \frac{20}{6EJ} + \overline{H_b}\left(\frac{20}{3EJ} + \frac{13}{3EJ}\right) + 150 \cdot \frac{13}{6EJ} - \frac{340}{EJ} - \frac{3670}{EJ} = 0$$

$\overline{H_b} = 198{,}6\,kNm$

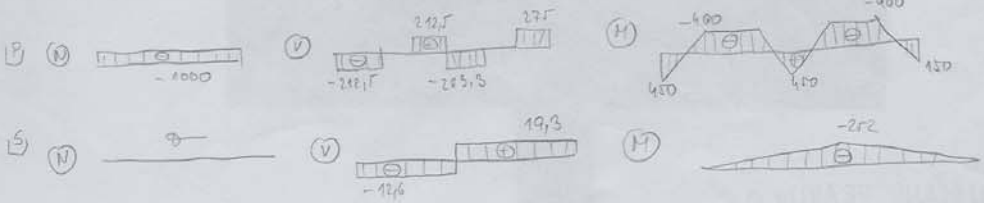
$20 \cdot R_a - 212{,}5 \cdot 20 + 450 + 212{,}5 \cdot 16 + 212{,}5 \cdot 4 - 198{,}6 = 0$

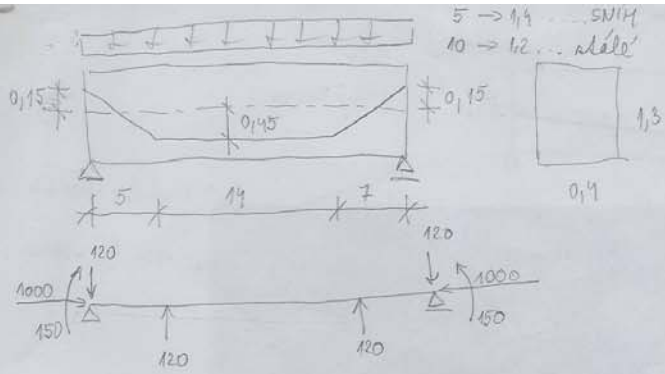
$13 \cdot R_c + 150 - 275 \cdot 13 + 11 \cdot 275 + 283{,}3 \cdot 3 - 198{,}6 = 0$

$R_a = -12{,}6\,kN\ (\downarrow)$

$R_c = -19{,}5\,kN\ (\downarrow)$

$\Big\} \; P_b = 31{,}9\,kN\ (\uparrow)$





3316

$\rho = 2,6 \cdot 10^3\ kg\,m^{-3} = 26\ kN\,m^{-3}$

$-0,5\ MPa$ v dolních vlákenech

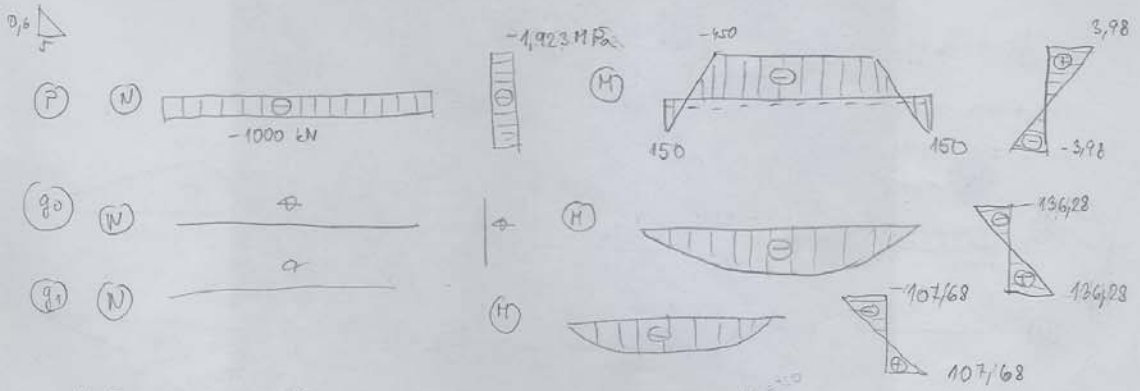
$A = 0,4 \cdot 1,3 = 0,52\ m^2$

$J = 0,0732\ m^4$

$W = 0,113\ m^3$

$g_0 = 13,52\ kN\,m^{-1}$

$g_1 = 12\ kN\,m^{-1}$

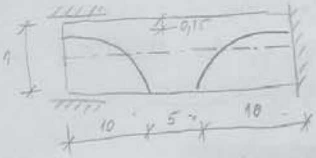


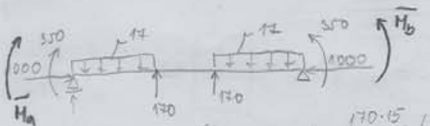
$\sigma_P + \sigma_{g0} + \sigma_{g1} = -0,5\ MPa$

$-1,923 - \sigma_P = -0,5 - 1,142 - 1,016 = -2,658 = m \cdot (-1,923) + m \cdot (-3,98)$

KOMBINAČNÍ PRAVIDLO

(7) SN 2X





$\varphi_{ab} = \frac{17 \cdot 10^2}{12EI}(3 \cdot 25 - 2 \cdot 10) - \frac{170 \cdot 15}{6EI \cdot 25}(25^2 - 15^2) - \frac{170 \cdot 10}{6EI \cdot 25}(25^2 - 10^2) =$

$= -\frac{14875}{3EI}$

$tg\alpha = \frac{2 \cdot 0{,}85}{10}$

$P_v \doteq 170 kN$

$p = \frac{8 \cdot 1000 \cdot 0{,}85}{20^2} = 17 kNm^{-1}$

$\varphi_{ba} = \frac{17 \cdot 10^2}{12EI}(3 \cdot 25 - 2 \cdot 10) - \frac{170 \cdot 10}{6EI \cdot 25}(25^2 - 10^2) - \frac{170 \cdot 15}{6EI \cdot 25}(25^2 - 15^2) = -\frac{14875}{3EI}$

$\overline{M_a}\frac{25}{3EI} + 350 \cdot \frac{25}{3EI} + 350 \cdot \frac{25}{6EI} + \overline{M_b} \cdot \frac{25}{6EI} - \frac{14875}{3EI} = 0$ $\Big] \cdot \left(-\frac{1}{2}\right)$

$\overline{M_a} \cdot \frac{25}{6EI} + 350 \cdot \frac{25}{6EI} + 350 \cdot \frac{25}{3EI} + \overline{M_b} \cdot \frac{25}{3EI} - \frac{14875}{3EI} = 0$

$350\left(\frac{25}{3EI} - \frac{25}{12EI}\right) + \overline{M_b}\left(\frac{25}{3EI} - \frac{25}{12EI}\right) + \frac{14875}{6EI} - \frac{14875}{3EI} = 0$

$25 \cdot R_a + 350 - 17 \cdot 10 \cdot 20 + \frac{140}{3} + 170 \cdot 15 + 170 \cdot 10 - 17 \cdot 10 \cdot 5 - 350 - \frac{140}{3} = 0$

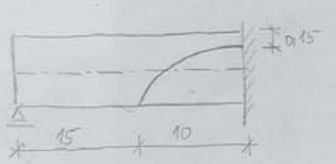
$\overline{M_b} = +\frac{140}{3} kNm$

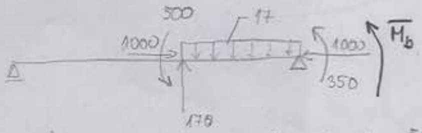
$\overline{M_a} = \frac{140}{3} kNm$

$R_a = R_b = 0$

---

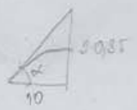
(8) 1xSN





$\varphi_{ba} = \frac{17 \cdot 10^2 \cdot 25}{6EI}\left(1 - \frac{10}{2 \cdot 25}\right)^2 - \frac{167{,}6 \cdot 15}{6EI \cdot 25}(25^2 - 15^2) - \frac{500 \cdot 25}{6EI}\left(1 - 3 \cdot \frac{15^2}{25^2}\right) =$

$= -\frac{2008}{EI}$



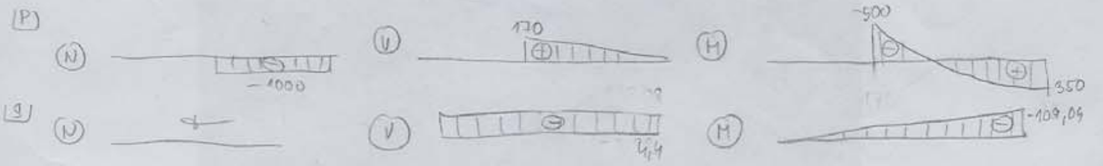
$tg\alpha = \frac{2 \cdot 0{,}85}{10} = 0{,}17$

$P_v = 167{,}6 kN$

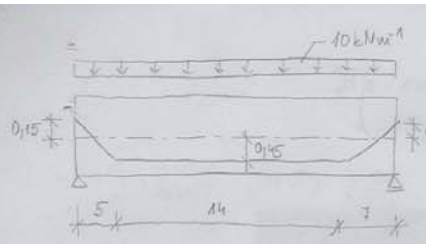
$p = \frac{8 \cdot 1000 \cdot 0{,}85}{20^2} = 17 kNm^{-1}$

$\overline{M_b} \cdot \frac{25}{3EI} + 350 \cdot \frac{25}{3EI} - \frac{2008}{EI} = 0 \Rightarrow \overline{M_b} = -109 kNm$

$25 R_a + 170 \cdot 10 - 17 \cdot 10 \cdot 5 - 500 - 350 + 109 = 0 \ldots R_a = -4{,}4 kN$



(c) nic jinde

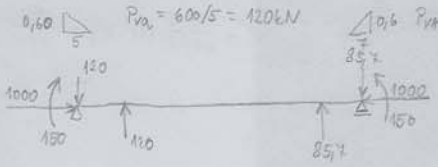


$\rho = 2600 kg\,m^{-3} = 26 kN m^{-3}$

$A = 0,52 m^2$ ..... $g_0 = A\,\gamma = \underline{13,52 kN m^{-1}}$

$W = \frac{1}{6} b h^2 = \frac{1}{6} \cdot 0,4 \cdot 1,3^2 = \underline{0,1127 m^3}$

$g_1 = 10 + 2 = 12 kN m^{-1}$

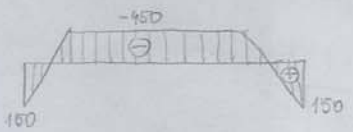


P (N)

$\sigma = \frac{N}{A} = 1,92 MPa$

$\sigma = \frac{M}{W} = 3,99 MPa$

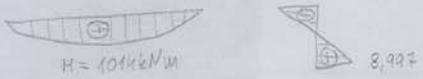
(M)



g_0 (M)

$\frac{1}{8} g l^2 = 1141,44 kNm$

(M)

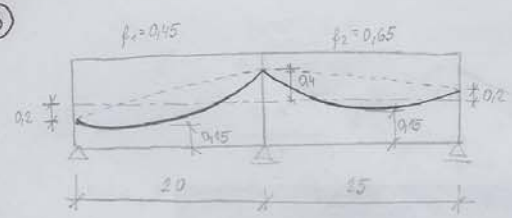


$\sigma_P + \sigma_{g0} + \sigma_{g1} = -0,5$ $\quad \sigma_P = \left| -0,5 - 10,137 - 8,997 \right| = -19,634 MPa = x\,\sigma_P = -1,92x - 3,99x \Rightarrow x \doteq 3,32$

$\Downarrow$

$P = 3320 kN$

(13)



$p = P_1 = 1000\,kN$

SN 1x ..... $x = \overline{M}_b$

$$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(y_2 - y_1)}{l} \cdot x$$

$$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{y_2 - y_1}{l}$$

$$y_1'(0) = \frac{4 \cdot 0{,}45}{20} + \frac{(-0{,}4 - 0{,}2)}{20} = 0{,}06 \ldots\ldots P_{1a} = P_{max} = 60\,kN$$

$$y_1'(20) = -\frac{8 \cdot 0{,}45 \cdot 20}{20^2} + \frac{4 \cdot 0{,}45}{20} - \frac{0{,}6}{20} = -0{,}12 \ldots\ldots P_{1b}^1 = 119\,kN$$

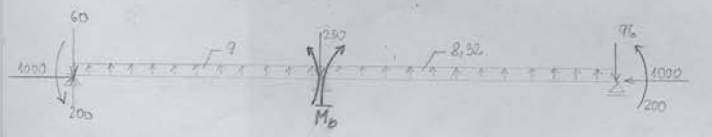
$$y_2'(0) = \frac{0{,}65 \cdot 4}{25} + \frac{(-0{,}2 + 0{,}4)}{25} = 0{,}112 \ldots\ldots P_{1b}^2 = 111\,kN$$

$P_{1b} = 230\,kN$

$$y_2'(25) = -\frac{8 \cdot 0{,}65 \cdot 25}{25^2} + \frac{4 \cdot 0{,}65}{25} + \frac{0{,}2}{25} = -0{,}096 \ldots\ldots P_{1c} = 96\,kN$$

$$p_1 = \frac{8 P_1 f_1}{l_1^2} = \frac{8 \cdot 1000 \cdot 0{,}45}{20^2} = 9\,kNm^{-1}$$

$$p_2 = \frac{8 P_1 f_2}{l_2^2} = \frac{8 \cdot 1000 \cdot 0{,}65}{25^2} = 8{,}32\,kNm^{-1}$$



$$\varphi_{1a} = \frac{1}{24}\frac{q_1 l_1^3}{EJ} = \frac{-9 \cdot 20^3}{24\,EJ} = \frac{-3000}{EJ}$$

$$\varphi_{1c} = \frac{1}{24}\frac{q_2 l_2^3}{EJ} = \frac{-8{,}32 \cdot 25^3}{24\,EJ} = -\frac{16250}{3EJ}$$

$$\varphi_{1a} + \varphi_{1c} + M_a \beta_{aa} + \overline{M}_b (\alpha_{ba} + \alpha_{bc}) + M_c \beta_{cb} = 0$$

$$-\frac{3000}{EJ} - \frac{16250}{3EJ} - 200 \cdot \frac{20}{6EJ} + \overline{M}_b \left(\frac{20}{3EJ} + \frac{25}{3EJ}\right) + 200 \cdot \frac{25}{6EJ} = 0$$

$\overline{M}_b = 550\,kNm$

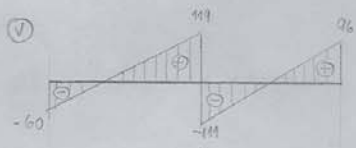
$$20 R_a - 60 \cdot 20 - 200 + \frac{1}{2} \cdot 9 \cdot 20^2 - 550 = 0$$

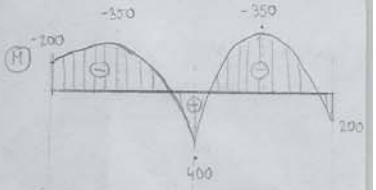
$R_a = 7{,}5\,kN\ (\uparrow)$

$$25 R_c + 200 - 96 \cdot 25 + \frac{1}{2} \cdot 8{,}32 \cdot 25^2 - 550 = 0$$

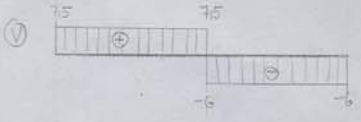
$R_c = 96\,kN\ (\uparrow)$ $\qquad$ $R_b = 185\,kN\ (\downarrow)$

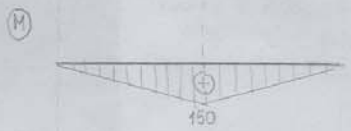
PRIMÁRNÍ ÚČINKY



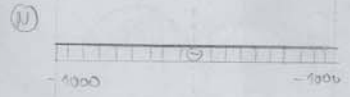


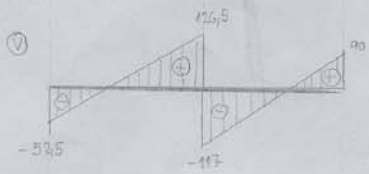
SEKUNDÁRNÍ ÚČINKY

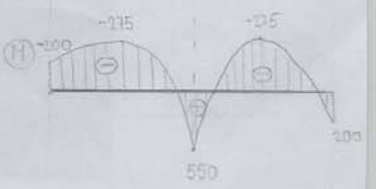




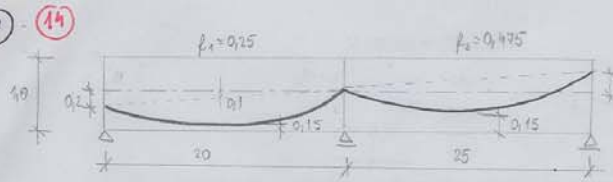
CELKOVÉ ÚČINKY







(14) - (14)



$P = P_H = 1000\,kN$

SN 1x → $X = \overline{H}_b$

$$y(x) = -\frac{4f_1x^2}{l^2} + \frac{4f_1x}{l} + \frac{(f_2-f_1)}{l}\cdot x$$

$$y'(x) = -\frac{8f_1}{l^2} + \frac{4f_1}{l} + \frac{f_2-f_1}{l}$$

$$y_1 = \frac{8\cdot P_H\cdot f_1}{l_1^2} = \frac{8\cdot 1000\cdot 0{,}125}{20^2} = 5\,kN\,m^{-1}$$

$$y_2 = \frac{8\cdot P_H\cdot f_2}{l_2^2} = \frac{8\cdot 1000\cdot 0{,}475}{25^2} = 6{,}08\,kN\,m^{-1}$$

$$y_1'(0) = \frac{4\cdot 0{,}125}{20} + \frac{0-0{,}2}{20} = 0{,}04 \ldots\ldots P_{Va} = 40kN$$

$$y_1'(20) = -\frac{8\cdot 0{,}125\cdot 20}{20^2} + \frac{4\cdot 0{,}125}{20} - \frac{0{,}2}{20} = -0{,}06 \ldots\ldots P_{Vb}^1 = 60kN$$

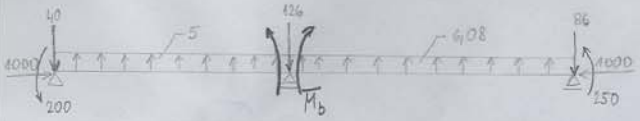
$$y_2'(0) = \frac{4\cdot 0{,}475}{25} + \frac{(-0{,}25-0)}{25} = 0{,}066 \ldots\ldots P_{Vb}^2 = 66\,kN$$

$P_{Vb} = 126kN$

$$y_2'(25) = -\frac{8\cdot 0{,}475\cdot 25}{25^2} + \frac{4\cdot 0{,}475}{25} - \frac{0{,}25}{25} = -0{,}086 \ldots\ldots P_{Vc} = 86\,kN$$

$$\varphi_{Ma} = \frac{1}{24}\frac{q\,l^3}{EJ} = \frac{-5\cdot 20^3}{24\,EJ} = -\frac{5000}{3EJ}$$

$$\varphi_{Mc} = \frac{1}{24}\frac{q\,l^3}{EJ} = \frac{-6{,}08\cdot 25^3}{24\,EJ} = -\frac{11875}{11\,EJ}$$



$$\varphi_{Ma} + \varphi_{Mc} + H_a\cdot\beta_{ab} + \overline{H}_b\,(\alpha_{Ma}+\alpha_{Mc}) + H_c\cdot\beta_{cb} = 0$$

$$-\frac{5000}{3EJ} - \frac{11875}{11EJ} - 200\cdot\frac{20}{6EJ} + \overline{H}_b\left(\frac{20}{3EJ}+\frac{25}{3EJ}\right) + 250\cdot\frac{25}{6EJ} = 0$$

$\overline{H}_b = 158\,kNm$

$$20\cdot R_a - 200 - 40\cdot 20 + \frac{1}{2}\cdot 5\cdot 20^2 - 158 = 0$$

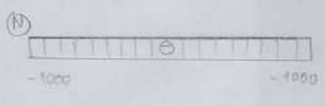
$R_a = 79kN\,(\uparrow)$

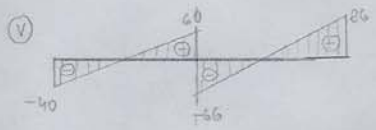
$$25\cdot R_c + 250 - 86\cdot 25 + \frac{1}{2}\cdot 6{,}08\cdot 25^2 - 158 = 0$$

$R_c = 6{,}92kN\,(\uparrow)$

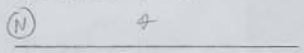
$R_c = 14{,}22kN\,(\downarrow)$

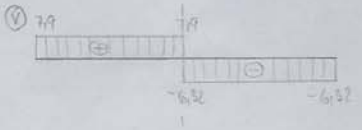
PRIMÁRNÍ ÚČINKY

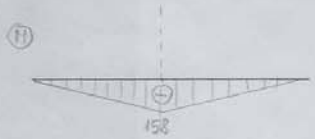




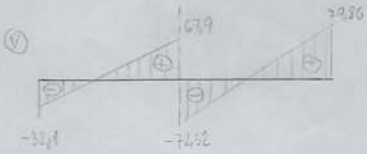
SEKUNDÁRNÍ ÚČINKY

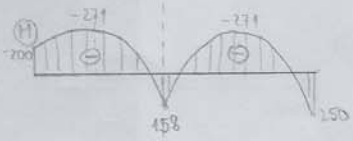




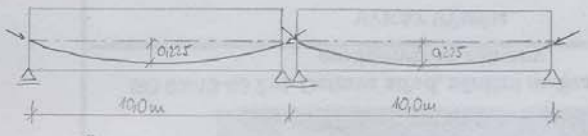


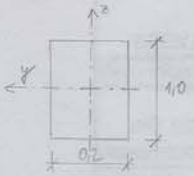
CELKOVÉ ÚČINKY





1. ETAPA $P_I = P_H = 1000\,kN$



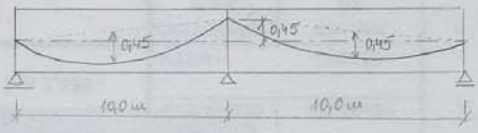


$A = 0,2 \cdot 1,0 = 0,2\,m^2$

$J = \frac{1}{12} \cdot 0,2 \cdot 1,0^3 = 0,166\,m^4$

$W = \frac{J}{z} = \frac{0,166}{0,5} = 0,033\,m^3$

2. ETAPA $P_{II} = P_H = 450\,kN$



$g_1 = 10\,kNm^{-1}$ (přibližně na spojitém nosníku)

$g_0 = 0,2 \cdot 25 = 5\,kNm^{-1}$

$v = ?$

• prostý nosník

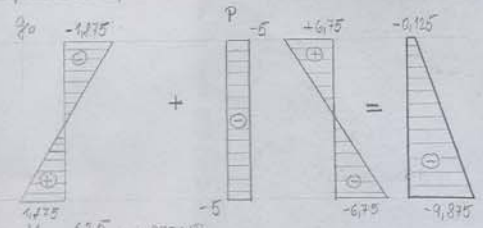
$M_{g_0} = \frac{1}{8} g_0 l^2 = \frac{1}{8} \cdot 5 \cdot 10^2 = 62,5\,kNm^{-1}$

- předpětí: $p = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0,225}{10^2} = 11\,kNm^{-1}$

$M_{P_I} = \frac{1}{8} p l^2 = \frac{1}{8} \cdot 11 \cdot 10^2 = 225\,kNm$

$N_{P_I} = 1000\,kN$

uprostřed pole

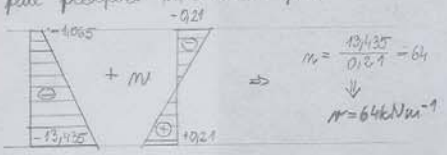


$\sigma = \frac{M}{W} = \frac{62,5}{0,033} = 1,875\,MPa$

$\sigma = \frac{N}{A} = \frac{1000}{0,2} = 5\,MPa$

$\sigma = \frac{M_{P_I}}{W} = \frac{225}{0,033} = 6,75\,MPa$

plus předpětí uprostřed pole

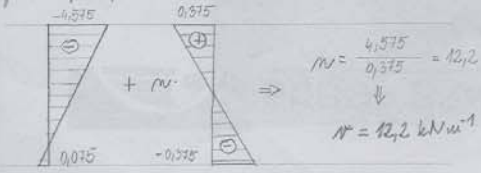


$\Rightarrow \quad v = \frac{13,435}{0,21} = 64$

$\Downarrow$

$v = 64\,kNm^{-1}$

plus předpětí nad podporou



$\Rightarrow \quad v = \frac{4,575}{0,375} = 12,2$

$\Downarrow$

$v = 12,2\,kNm^{-1}$

• spojitý nosník

$M_{g_1}(pole) = 0,0703 g_1 l^2 = 0,0703 \cdot 10 \cdot 10^2 = 70,3\,kNm$ (tah)

$M_{g_1}(podpora) = \frac{1}{8} g_1 l^2 = \frac{1}{8} \cdot 10 \cdot 10^2 = 125\,kNm$ (tlak)

- předpětí: $M_t = \frac{8 P_H f}{l^2} = \frac{8 \cdot 450 \cdot 0,45}{10^2} = 16,2\,kNm^{-1}$

$M_{P_H}(pole) = 0,0703 \cdot 16,2 \cdot 10^2 = 113,9\,kNm$

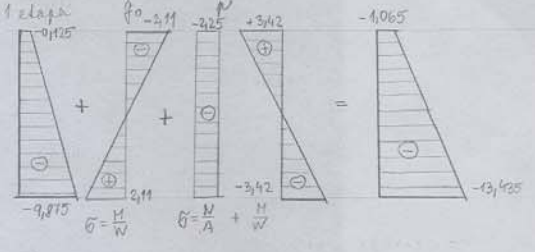
$M_{P_H}(podpora) = \frac{1}{8} \cdot 16,2 \cdot 10^2 = 202,5\,kNm$

- zatížení: $v = 1\,kNm^{-1}$

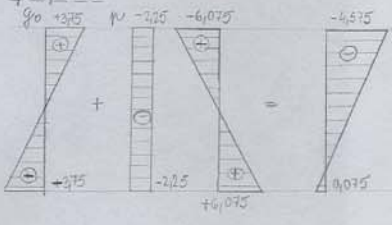
$M_v(pole) = 0,0703 \cdot 1 \cdot 10^2 = 7,03\,kNm$

$M_v(podpora) = \frac{1}{8} \cdot 1 \cdot 10^2 = 12,5\,kNm$

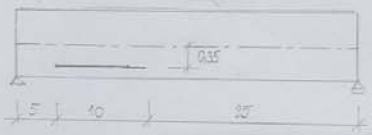
uprostřed pole



podpora



②



SN 0x

$P = P_H = 1000\,kN$

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

(N) 

(V) 

(H) 

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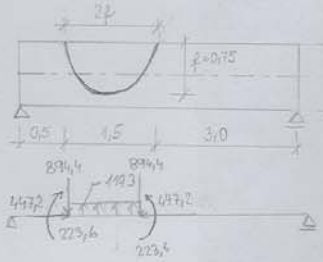
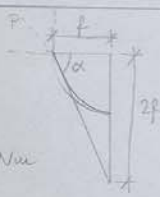
③



SN 0x

1,0 · P = 1000 kN

$M = P_H \cdot 0{,}5 = 223{,}6\,kNm$



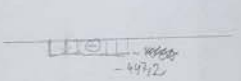
$tg\,\alpha = \frac{2f}{l} = 2$

$P_V = P \cdot \sin\alpha = 894{,}4\,kN$

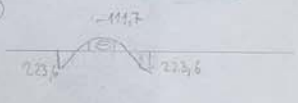
$P_H = P \cdot \cos\alpha = 447{,}2\,kN$

$\mu = \frac{8 \cdot P_H \cdot f}{l_p^2} = \frac{8 \cdot 447{,}2 \cdot 0{,}75}{1{,}5^2} = 1193\,kNm^{-1}$

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

(N) 

(V) 

(M) 

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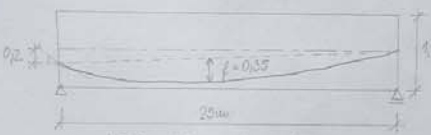
④



SN 0x

$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(x_2 - x_1)\cdot x}{l}$

$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{x_2 - x_1}{l}$

$y'(0) = \frac{4 \cdot 0{,}35}{25} + \frac{0 - 0{,}2}{25} = tg\,\alpha_1 = 0{,}048$

$y'(25) = -\frac{8 \cdot 0{,}35 \cdot 25}{25^2} + \frac{4 \cdot 0{,}35}{25} - \frac{0{,}2}{25} = tg\,\alpha_2 = -0{,}064$

$\varphi_{0E} = \varphi_{0A} = \frac{1}{24}\frac{p l^3}{EJ} = \frac{1}{24}\frac{(-4{,}48)\cdot 25^3}{EJ} = \frac{10000}{24EJ}$

$P_V^1 = P \cdot \sin\alpha_1 = 47{,}9\,kN \qquad P_H^1 = 998{,}8\,kN$

$P_V^2 = P \cdot \sin\alpha_2 = 63{,}9\,kN \qquad P_H^2 = P \cdot \cos\alpha_2 = 998{,}0\,kN$

$\Big\} P_H = 1000\,kN$

$M = P_H^1 \cdot 0{,}2 = 179{,}76\,kNm \doteq 200\,kNm$

$\mu = \frac{8 \cdot P_H \cdot f}{l^2} = \frac{8 \cdot 1000 \cdot 0{,}35}{25^2} = 4{,}48\,kNm^{-1}$

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

(N) 

(V) 

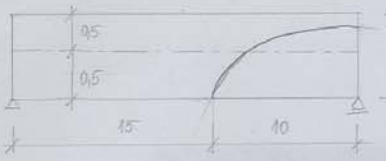
(M) 

$M(l/2) = 200 + 48 \cdot \frac{25}{2} - \frac{1}{8} \cdot 4{,}48 \cdot 25^2 = 450\,kNm$

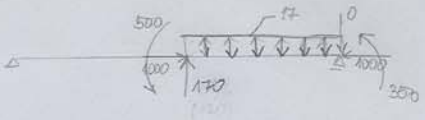
M

⑤

SN 0x



P = 1000 kN



PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

$tg\alpha = \frac{2 \cdot 0,85}{10} = 0,17$

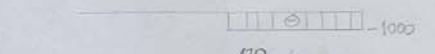
$P_H = P = 1000\,kN$ $P_V = P \cdot \sin\alpha = 167,6\,kN \doteq 170\,kN$

$M^1 = 0,5 \cdot P_H = 500\,kNm$ $M^2 = 0,35 \cdot P_H = 350\,kNm$

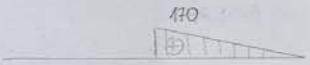
$p = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0,85}{20^2} \doteq 17,0\,kNm^{-1}$

$M(4/2) = 500 - 167,6 \cdot 5 + \frac{1}{8} \cdot 17 \cdot 10^2 = -550,5\,kNm$

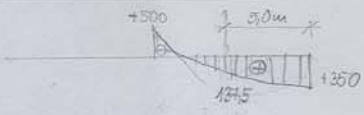
(N)



(V)



(M)



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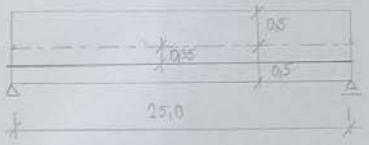
⑥

SN 0x



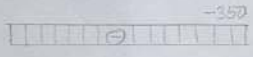
P = 1000 kN

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

(N)

(V)

(M)



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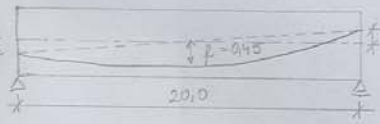
⑦

SN 0x



P = P_H = 1000 kN

$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(e_2 - e_1)x}{l}$

$y'(x) = -\frac{8fx}{l} + \frac{4f}{l} + \frac{e_2 - e_1}{l}$

$y'(0) = \frac{4 \cdot 0,45}{20} + \frac{-0,35 - 0,15}{20} = tg\alpha_1 = 0,065$

$y'(20) = -\frac{8 \cdot 0,15 \cdot 20}{20^2} + \frac{4 \cdot 0,45}{20} - \frac{0,6}{20} = tg\alpha_2 = -0,115$

$P_V^1 = P \cdot \sin\alpha_1 \doteq 65,0\,kN$

$P_V^2 = P \cdot \sin\alpha_2 \doteq 114\,kN$

$M^1 = 1000 \cdot 0,15 = 150\,kNm$

$M^2 = 1000 \cdot 0,35 = 350\,kNm$

$p = \frac{8 \cdot P_H \cdot f}{l^2} = \frac{8 \cdot 1000 \cdot 0,45}{20^2} = 9\,kNm^{-1}$

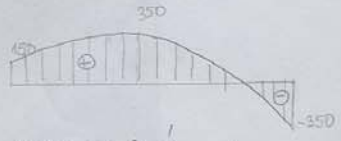
$M(9/2) = 150 + 65 \cdot 10 - \frac{1}{8} \cdot 9 \cdot 20^2 =$

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

(N)

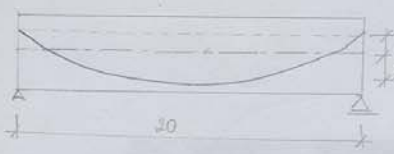
(V)

(M)



FILIP: OPAČNÁ ZNAMÉNKA

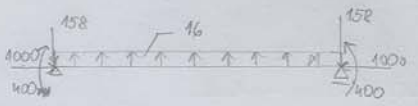
⑧



SN OX

P = 1000 kN

$\doteq P_H$



$y_z(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(z_2 - z_1)x}{l}$

$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{z_2 - z_1}{l}$

$y'(0) = -y'(20) = \operatorname{tg}\alpha = \frac{4 \cdot 0{,}8}{20} + \frac{(-0{,}4+0{,}4)}{20} = 0{,}16$

$P_V = P \cdot \sin\alpha \doteq 158\,kN$

$p = \frac{8 \cdot P_H \cdot f}{l^2} = \frac{8 \cdot 1000 \cdot 0{,}8}{20^2} = 16\,kN\,m^{-1}$

$M(l/2) = 400 - 158 \cdot 10 + \frac{1}{8} \cdot 16 \cdot 20^2 = -380\,kN\,m$

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

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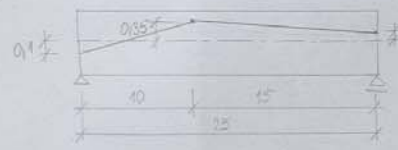
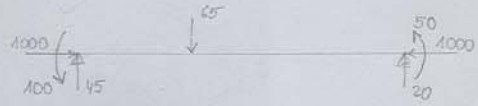
⑨ *



SN OX

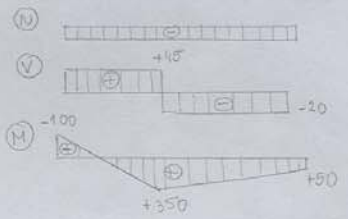
P = 1000 kN



PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

$\frac{0{,}45}{10} = \frac{P_V^1}{1000}$ $\quad P_V^1 = 45\,kN$

$\frac{0{,}30}{15} = \frac{P_V^2}{1000}$ $\quad P_V^2 = 20\,kN$



$M(x) =$

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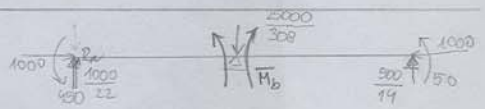
⑨ ✓

SN 1x

$X = \overline{M}_b$

P = 1000 kN



$\frac{1}{22} = \frac{P_V^1}{1000} \rightarrow P_V^1 = \frac{1000}{22}$ $\quad \frac{0{,}5}{14} = \frac{P_V^2}{1000} \rightarrow P_V^2 = \frac{500}{14}$

$M_a \cdot f_{ab} + \overline{M}_b (\alpha_{ba} + \alpha_{bc}) + M_c \cdot f_{cb} = 0$

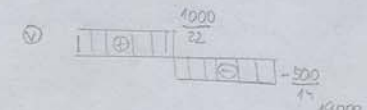
$450 \cdot \frac{22}{6EJ} + \overline{M}_b \left(\frac{22}{3EJ} + \frac{14}{3EJ} \right) + 50 \cdot \frac{14}{6EJ} = 0$

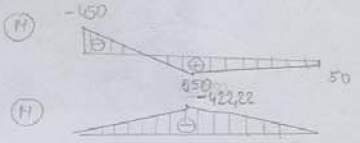
$22 \cdot R_a + \frac{1000}{22} \cdot 22 - 450 - \frac{1150}{9} = 0 \quad \ldots R_a = + \frac{19000}{990}\,kN\,(\downarrow)$

$\frac{36}{3EJ} \overline{M}_b = -\frac{9200}{6EJ} \quad \ldots \overline{M}_b = -\frac{1150}{9}\,kNm$

$14 \cdot R_c + \frac{500}{14} \cdot 14 + 50 - \frac{1150}{9} = 0 \quad \ldots R_c = \frac{19000}{630}\,kN\,(\downarrow)$

PRIMÁRNÍ ÚČINKY





SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

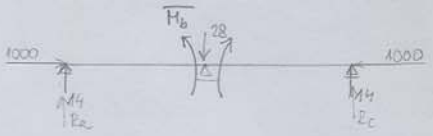
✓ (10)

SN 1x

$\Downarrow$

$X = \overline{M_b}$

$P = 1000\,kN$



$M_a \cdot \beta_{aa} + \overline{M_b}\,(\alpha_{ba} + \alpha_{bc}) + M_c \cdot \beta_{cb} = 0$

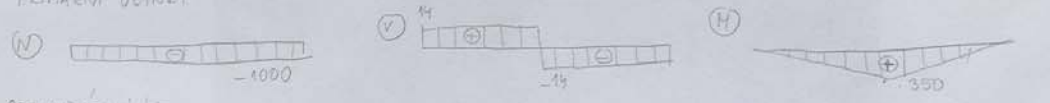
$\Rightarrow \overline{M_b} = 0$

$P_V^a = P_V^2 = 1000 \cdot \frac{0,35}{25} = 14\,kN$

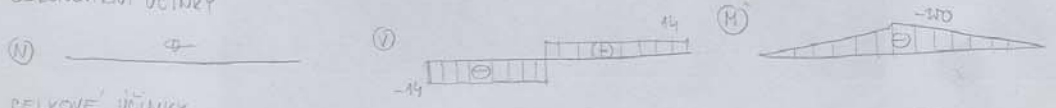
$R_a: 25 + 14 \cdot 25 = 0 \Rightarrow R_a = -14\,kN\ (\downarrow)$

$R_c = -14\,kN\ (\downarrow)$

PRIMÁRNÍ ÚČINKY

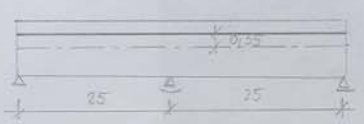


SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY

✓ (11)



SN 1x

$\Downarrow$

$X = \overline{M_b}$

$P = 1000\,kN$

$M_a \beta_{aa} + \overline{M_b}\,(\alpha_{ba} + \alpha_{bc}) + M_c \cdot \beta_{cb} = 0$

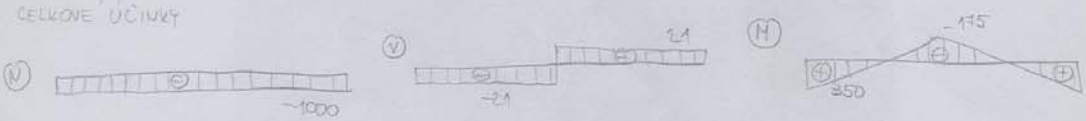
$350 \cdot \frac{25}{6EJ} + \overline{M_b} \cdot \left(\frac{25}{3EJ} + \frac{25}{3EJ}\right) + 350 \cdot \frac{25}{6EJ} = 0 \quad \dots \quad \frac{50}{3EJ}\,\overline{M_b} = -\frac{17500}{6EJ} \Rightarrow \overline{M_b} = -175\,kNm$

$25 R_a + 350 + 875 = 0 \quad \dots \quad R_a = R_c = -21\,kN\ (\downarrow)$

PRIMÁRNÍ ÚČINKY

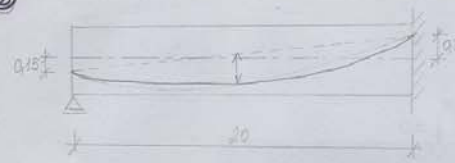
SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



✓

(12)



$\rho = 0{,}45$

SN 1x

$X = \overline{H}_b$

$P = P_H = 100\,kN$

$$\mu = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0{,}45}{20^2} = 9\,kNm^{-1}$$

$$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(y_2 - y_1)}{l} x$$

$$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{y_2 - y_1}{l}$$

$P_V^1 = P \cdot \sin\alpha_1 = 65\,kN$

$P_V^2 = P \cdot \sin\alpha_2 = 114\,kN$

$$\varphi_{ba} = \frac{1}{24}\frac{l^3}{EJ} - \frac{1}{24}\frac{-9 \cdot 20^3}{EJ} = \frac{-3000}{EJ}$$

$$y'(0) = \operatorname{tg}\alpha_1 = \frac{4 \cdot 0{,}45}{20} + \frac{(-0{,}35 - 0{,}10)}{20} = 0{,}065$$

$$y'(20) = \operatorname{tg}\alpha_2 = -\frac{8 \cdot 0{,}45 \cdot 20}{20^2} + \frac{4 \cdot 0{,}45}{20} - \frac{0{,}5}{20} = -0{,}115$$

$$\varphi_{ba} + M_a \beta_{ab} + H_b \alpha_{ba} + \overline{H}_b \alpha_{bb} = 0$$

$$-\frac{3000}{EJ} - 150 \cdot \frac{20}{6EJ} + 350 \cdot \frac{20}{3EJ} + \overline{H}_b \frac{80}{3EJ} = 0$$

$\overline{H}_b = 175\,kNm$

$150 + 65 \cdot 20 - 20 P_a - \frac{1}{2} \cdot 9 \cdot 20^2 + 350 + 175 = 0 \ldots P_a = 8{,}75\,kN\ (\uparrow)$

$P_b = 8{,}75\,kN\ (\downarrow)$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

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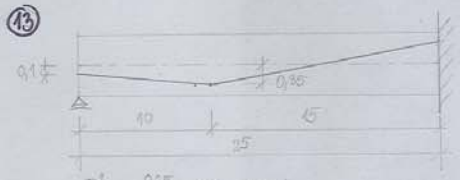
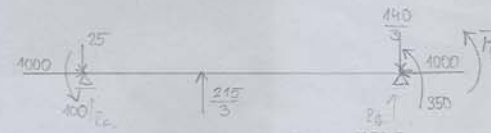
(13)



SN 1x

$X = \overline{H}_b$

$P = P_H = 1000\,kN$



$$\varphi_{ba} = \frac{F \cdot a}{6EJL}\left(L^2 - a^2\right) = -\frac{215}{3} \cdot \frac{10}{6EJ25}\left(25^2 - 10^2\right) = -\frac{7625}{3EJ}$$

$P_V^1 = \frac{0{,}25}{10} \cdot 1000 = 25\,kN$

$P_V^2 = \frac{0{,}10}{15} \cdot 1000 = \frac{140}{3}\,kN$

$$\varphi_{ba} + M_a \beta_{ab} + H_b \alpha_{ba} + \overline{H}_b \alpha_{bb} = 0$$

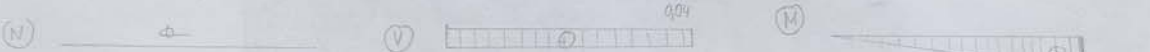
$$-\frac{7625}{3EJ} - 100 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{3EJ} = 0 \ldots \overline{H}_b = 1\,kNm$$

$25 P_a - 100 - 25 \cdot 25 + \frac{215}{3} \cdot 15 - 350 - 1 = 0$

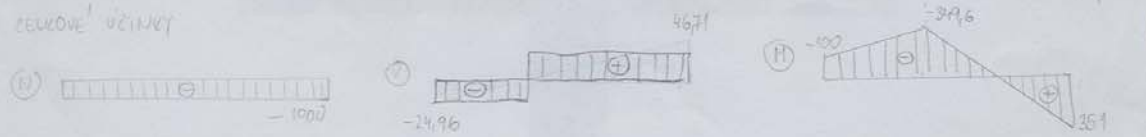
$P_a = 0{,}04\,kN\ (\uparrow)$ $\quad P_b = 0{,}04\,kN\ (\downarrow)$

PRIMÁRNÍ ÚČINKY

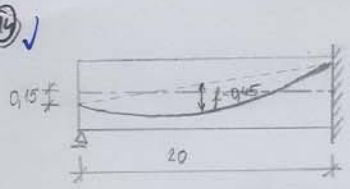
SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY



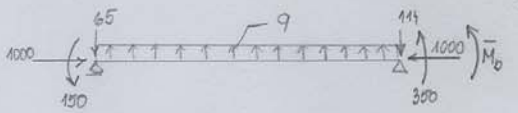
14)



SN 1x

$X = \overline{M}_b$

$P = P_H = 1000\,kN$



$$y(x) = -\frac{4fx^2}{l^2} + \frac{4fx}{l} + \frac{(e_2-e_1)x}{l}$$

$$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{e_2-e_1}{l}$$

$P_v = P \sin\alpha$

$$y'(0) = \operatorname{tg}\alpha_1 = \frac{4\cdot 0{,}45}{20} + \frac{-0{,}35-0{,}15}{20} = 0{,}065\ldots \quad P_v^1 = 65\,kN$$

$$y'(20) = \operatorname{tg}\alpha_2 = -\frac{8\cdot 0{,}45\cdot 20}{20^2} + \frac{4\cdot 0{,}45}{20} - \frac{0{,}5}{20} = -0{,}115\ldots \quad P_v^2 = 114\,kN$$

$$p = \frac{8Pf}{l^2} = \frac{8\cdot 1000\cdot 0{,}45}{20^2} = 9\,kNm^{-1}$$

$$\varphi_{ba} = \frac{1}{24}\frac{pl^3}{EJ} = \frac{1}{24}\cdot\frac{-9\cdot 20^3}{EJ} = -\frac{3000}{EJ}$$

$$M_a\cdot\beta_{ab} + \varphi_{ba} + M_b\cdot\alpha_{ba} + \overline{M}_b\cdot\alpha_{ba} = 0$$

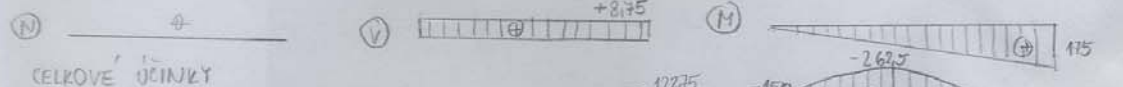
$$-150\cdot\frac{20}{6EJ} - \frac{3000}{EJ} + 350\cdot\frac{20}{3EJ} + \overline{M}_b\cdot\frac{20}{3EJ} = 0$$

$\overline{M}_b = 175\,kNm$ $\quad Z_a = 8{,}75\,kN\ (\uparrow)$

$R_b = 8{,}75\,kN\ (\downarrow)$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY

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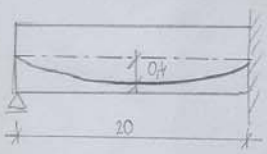
15)



SN 1x

$X = \overline{M}_b$

$P = P_H = 1000\,kN$

$$y'(0) = y'(20) = \frac{4\cdot 0{,}4}{20} = 0{,}08\ldots \quad P_v = 80\,kN$$

$$20R_a - 80\cdot 20 + \frac{1}{2}\cdot 8\cdot 20^2 - 400 = 0$$

$R_a = -R_b = 20\,kN$

$$p = \frac{8\cdot 1000\cdot 0{,}4}{20^2} = 8\,kNm^{-1}$$

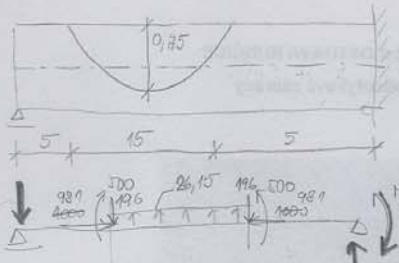
$$\varphi_{ba} = \frac{1}{24}\cdot\frac{-8\cdot 20^3}{EJ} = \frac{-8000}{3EJ} \qquad \frac{-8000}{3EJ} + \overline{M}_b\cdot\frac{20}{3EJ} = 0$$

$\overline{M}_b = 400\,kNm$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



$\operatorname{tg}\alpha = \frac{1,5}{7,5} = 0,2$

$P_V = P \cdot \sin\alpha = 196\,kN$

$P_H = P \cdot \cos\alpha = 981\,kN$

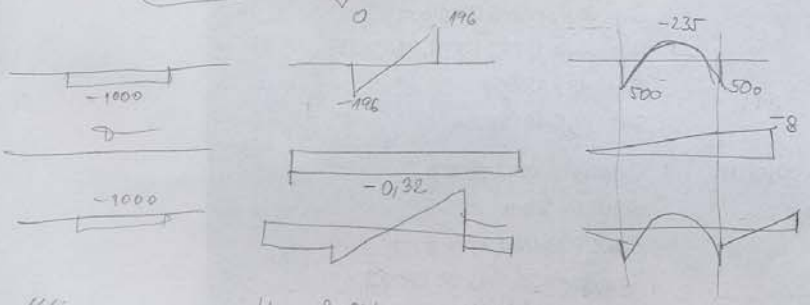
$p = \frac{3 \cdot 981 \cdot 0,75}{15^2} = 26,13\,kNm^{-1}$

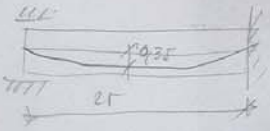
$$\varphi_{ba} = \frac{500 \cdot 25}{6EJ}\left(1 - 3 \cdot \frac{5^2}{25^2}\right) - \frac{500 \cdot 25}{6EJ}\left(1 - 3 \cdot \frac{20^2}{25^2}\right) - \frac{26,13}{24EJ} \cdot \frac{12,5 \cdot 15}{25}\left[4 \cdot 12,5\,(25 + 12,5) - 15^2\right] +$$

$$+ \frac{\cancel{8}}{3J} + \frac{196 \cdot 5}{2EJ}(25 - 5) = \frac{66,40625}{EJ}$$

$$M_b \cdot \frac{25}{3EJ} + \frac{66,40625}{EJ} = 0 \Rightarrow M_b = -8\,kNm$$

$25 \cdot R_a - 196 \cdot 20 - 196 \cdot 5 + 26,13 \cdot 15 \cdot 12,5 + 8 = 0$ $\quad R_a = -R_b = -0,32\,kN$



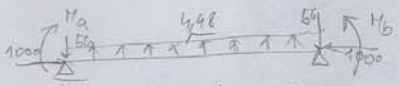


2x SN

$P_V = 56\,kN$

$P_H = 1000\,kN$

$\operatorname{tg}\alpha = 0,056$



$$M_a \cdot \frac{25}{3EJ} - \frac{8750}{3EJ} + M_b \cdot \frac{25}{6EJ} = 0 \quad | \cdot \left(-\frac{1}{2}\right)$$

$$M_a \cdot \frac{25}{6EJ} - \frac{8750}{3EJ} + M_b \cdot \frac{25}{3EJ} = 0$$

$$\varphi_{ab} = -\frac{1}{24}\frac{4,48 \cdot 25^3}{EJ} = -\frac{8750}{3EJ}$$

$$\frac{8750}{6EJ} - \frac{8750}{3EJ} + M_b \cdot \left(\frac{25}{3EJ} - \frac{25}{12EJ}\right) = 0$$

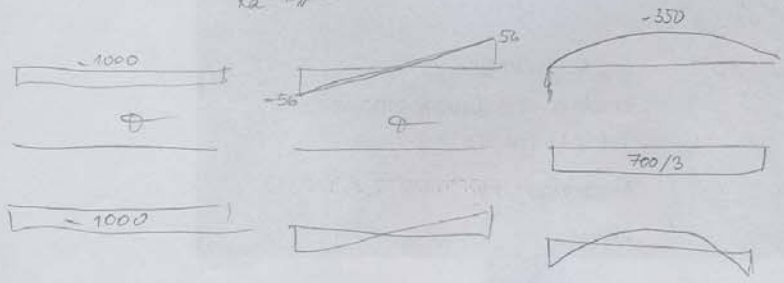
$= \varphi_{ba}$

$$M_b \frac{75}{12EJ} \Rightarrow M_b = 233\,kNm = \frac{700}{3}$$

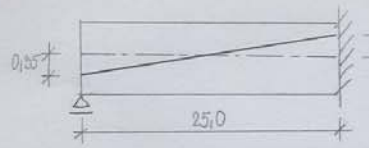
$M_a = M_b$

$$25\,R_a + \frac{700}{3} - 56 \cdot 25 + \frac{1}{2} \cdot 4,48 \cdot 25^2 - \frac{700}{3} = 0$$

$R_a = R_b = 0$



(16)



SN 1x

$\bar{M}_b = X$

$P_H = 1000\,kN$

$P_V = 1000 \cdot \frac{0,7}{25} = 28\,kN$

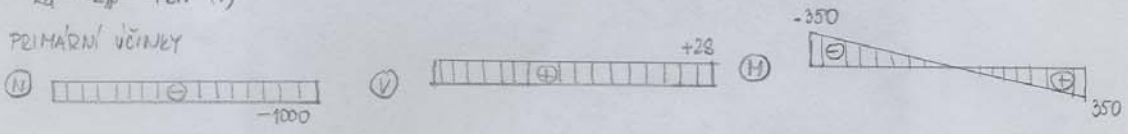
$-350 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \bar{M}_b \cdot \frac{25}{3EJ} = 0$

$\bar{M}_b = -175\,kNm$

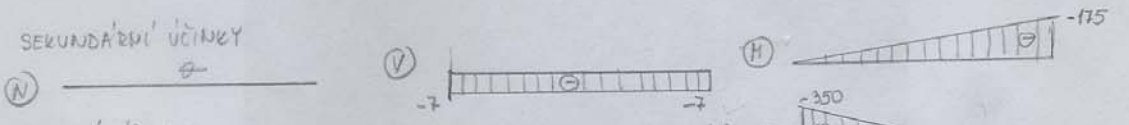
$25\,R_a + 28 \cdot 25 - 350 + 175 - 350 = 0$

$R_a = -R_b = -7\,kN \;(\downarrow)$

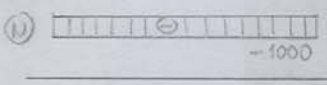
PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY



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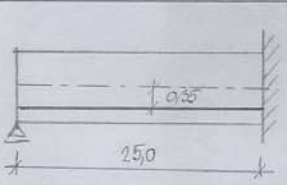
(17)



SN 1x

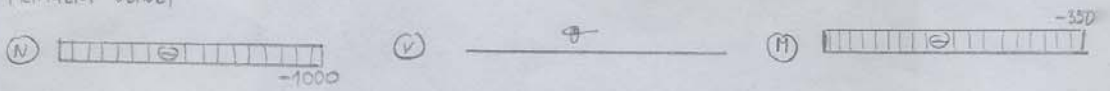
$X = M_b$

$P = 1000\,kN$

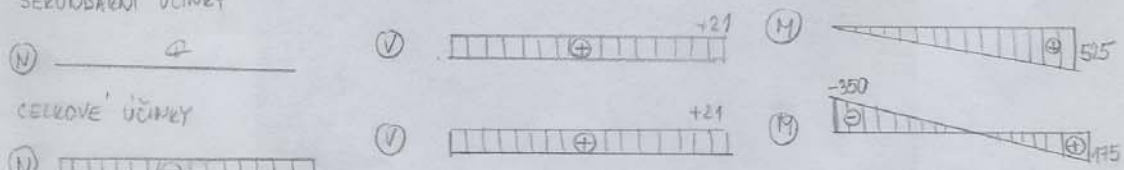
$-350 \cdot \frac{25}{6EJ} - 350 \cdot \frac{25}{3EJ} + \bar{M}_b \cdot \frac{25}{3EJ} = 0 \qquad \bar{M}_b = 525\,kNm$

$-350 + 25\,R_a + 350 - 525 = 0 \qquad R_a = -R_b = 21\,kN$

PRIMÁRNÍ ÚČINKY

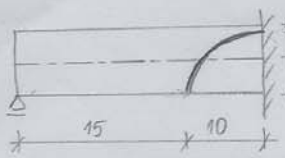


SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY

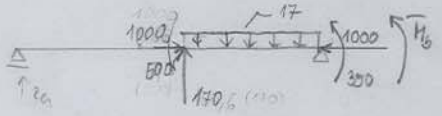
(20)



SN 1x

$X = \overline{M}_b$

$P = 1000\,kN$



$$\varphi_{ba} = \frac{+17 \cdot 10^2 \cdot 25}{6EJ}\left(1 - \frac{10}{2 \cdot 25}\right)^2 - \frac{167{,}6 \cdot 15^4}{25 \cdot 6EJ} - \frac{493 \cdot 25}{6EJ}\left(1 - 3 \cdot \frac{15^2}{25^2}\right) =$$

$$= -\frac{6019}{3EJ}$$

$\operatorname{tg}\alpha = \frac{1{,}7}{10} = 0{,}17$

$P_V = P \cdot \sin\alpha = 167{,}6\,kN \doteq 170\,kN$

$P_H = P \cdot \cos\alpha = 986\,kN \doteq 1000\,kN$

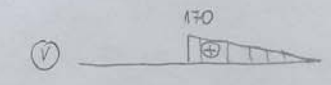
$$350 \cdot \frac{25}{3EJ} - \frac{6019}{3EJ} + \overline{M}_b \cdot \frac{25}{3EJ} = 0 \qquad \overline{M}_b = -109{,}24\,kNm$$

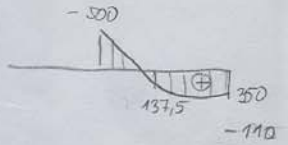
$$q = \frac{8 P_H f}{l^2} = \frac{8 \cdot 986 \cdot 0{,}85}{10^2} = 17\,kNm^{-1}$$

$$25 \cdot R_a + 167{,}6 \cdot 10 - 493 - \tfrac{1}{2} \cdot 17 \cdot 10^2 - 350 + 109{,}24 = 0$$

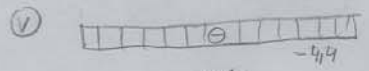
$R_a = -4{,}4\,kN \qquad -R_b$

PRIMÁRNÍ ÚČINKY

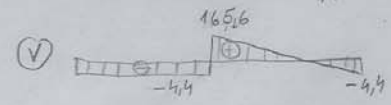




SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY



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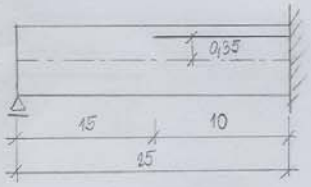
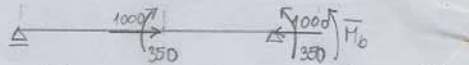


SN 1x

$X = \overline{M}_b$

$P_H = 1000\,kN$

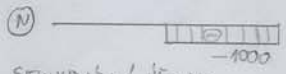


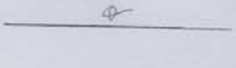
$$\varphi_{ba} = \frac{M l}{6EJ}\left(1 - 3\frac{a^2}{l^2}\right) = \frac{350 \cdot 25}{6EJ}\left(1 - 3 \cdot \frac{15^2}{25^2}\right) = -\frac{350}{3EJ}$$

$$\frac{-350}{3EJ} + \overline{M}_b \frac{25}{3EJ} + 350 \cdot \frac{25}{3EJ} = 0 \qquad \overline{M}_b = -336\,kNm$$

PRIMÁRNÍ ÚČINKY

$$25 \cdot R_a + 350 - 350 + 336 = 0 \qquad R_a = -13{,}44\,kN = -R_b$$

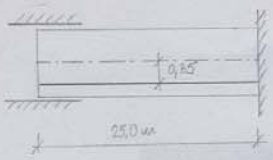




SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

21.



$P = 1000\,kN$

SN 2x

$X_1 = \overline{H}_a$

$X_2 = \overline{H}_b$

$\overline{H}_a \cdot \alpha_{at} + H_a \cdot \alpha_{at} + \overline{H}_b \cdot \beta_{ba} + \overline{H}_b \cdot \beta_{ba} = 0$

$\overline{H}_a \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{6EJ} = 0$

$\overline{H}_a \cdot \frac{25}{3EJ} - \frac{4375}{EJ} + \overline{H}_b \cdot \frac{25}{6EJ} = 0 \quad / \cdot (-\frac{1}{2})$

$\frac{1}{2} \cdot \frac{4375}{EJ} - \frac{4375}{EJ} - \overline{H}_b \frac{25}{12EJ} + \overline{H}_b \cdot \frac{25}{3EJ} = 0$

$\overline{H}_b = -350\ kNm$

$\overline{H}_a = 350\ kNm$

$\overline{H}_a \cdot \beta_{ab} + H_a \cdot \beta_{ab} + \overline{H}_b \cdot \alpha_{ba} + \overline{H}_b \cdot \alpha_{ba} = 0$

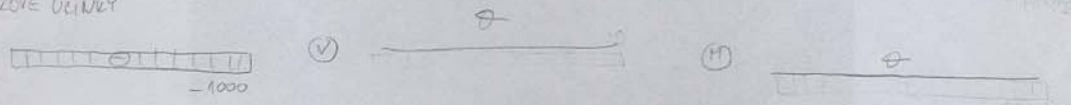
$\overline{H}_a \cdot \frac{25}{6EJ} - \frac{4375}{EJ} + \overline{H}_b \cdot \frac{25}{3EJ} = 0$

$25 \cdot R_a - 350 + 350 = 0 \quad \dots \quad R_a = R_b = 0\,kN$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



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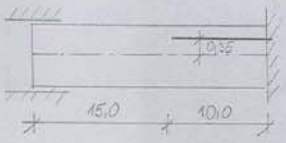
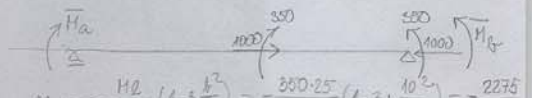


$P = 1000\,kN$

$2 \times SN$

$X_1 = \overline{H}_a \quad X_2 = \overline{H}_b$



$\varphi_{at} = -\frac{Ml}{6EJ}\left(1-3\frac{b^2}{l^2}\right) = -\frac{350 \cdot 25}{6EJ}\left(1-3 \cdot \frac{10^2}{25^2}\right) = -\frac{2275}{3EJ}$

$\varphi_{ba} = \frac{Ml}{6EJ}\left(1-3\frac{a^2}{l^2}\right) = \frac{350 \cdot 25}{6EJ}\left(1-3 \cdot \frac{15^2}{25^2}\right) = -\frac{350}{3EJ}$

$\overline{H}_a \cdot \alpha_{at} + H_a \cdot \beta_{ba} + \overline{H}_b \cdot \beta_{ba} + \varphi_{at} = 0$

$\overline{H}_a \cdot \beta_{at} + H_a \cdot \alpha_{ba} + \overline{H}_b \cdot \alpha_{ba} + \varphi_{ba} = 0$

$\overline{H}_a \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{6EJ} - \frac{2275}{3EJ} = 0 \quad / \cdot (-\frac{1}{2})$

$\overline{H}_a \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{3EJ} - \frac{350}{3EJ} = 0$

$-\frac{1350}{EJ} - \overline{H}_b \cdot \frac{25}{12EJ} + \frac{2800}{EJ} + \overline{H}_b \cdot \frac{25}{3EJ} = 0$ Mlost

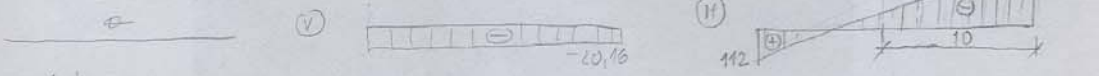
$\overline{H}_b = -392\,kNm \quad \overline{H}_a = 112\,kNm$

$25\,R_a + 112 + 350 - 350 + 392 = 0 \quad \dots \quad R_a = -R_b = -20,16\,kN$

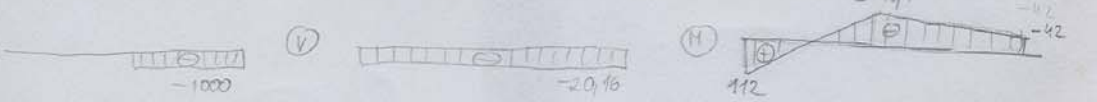
$M_a = 238\,kNm \quad M_b = 308\,kNm$ chyba

PRIMÁRNÍ ÚČINKY

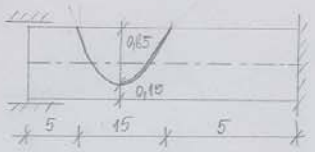
SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY



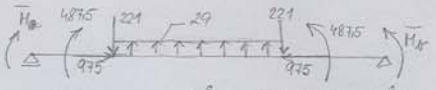
(23)



$P = 1000\,kN$

$2\times SN$

$X_1 = \overline{M}_a \quad X_2 = \overline{M}_b$



$tg\alpha = \frac{17}{15/2} = 0{,}23 \qquad p = \frac{8 P_H f}{l^2} = 29{,}5\,kN\,m^{-1}$

$\left(\frac{2 \cdot 975 \cdot 0{,}85}{15^2}\right)$

$P_V = P_{max} \cdot \sin\alpha = 221\,kN$

$P_H = P \cdot \cos\alpha = 975\,kN$

$$\varphi_{ab} = -\frac{4875 \cdot 25}{6EJ}\left(1 - 3\cdot\frac{20^2}{25^2}\right) + \frac{4875 \cdot 25}{6EJ}\left(1 - 3\cdot\frac{5^2}{25^2}\right) - \frac{29}{24EJ}\cdot\frac{12{,}5 \cdot 15}{25}\left[4 \cdot 12{,}5(25 + 12{,}5) - 15^2\right] + \frac{221 \cdot 5}{2EJ}(25 - 5) = -\frac{246{,}875}{EJ}$$

$$\varphi_{ba} = \frac{4875 \cdot 25}{6EJ}\left(1 - 3\cdot\frac{5^2}{25^2}\right) - \frac{4875 \cdot 25}{6EJ}\left(1 - 3\cdot\frac{20^2}{25^2}\right) - \frac{29}{24EJ}\cdot\frac{12{,}5 \cdot 15}{25}\left[4 \cdot 12{,}5(25 + 12{,}5) - 15^2\right] + \frac{221 \cdot 5}{2EJ}(25 - 5) = -\frac{246{,}875}{EJ}$$

$$\overline{M}_a \cdot \frac{25}{3EJ} + \overline{M}_b \cdot \frac{25}{6EJ} - \frac{246{,}875}{EJ} = 0 \quad / \cdot(-\tfrac{1}{2})$$

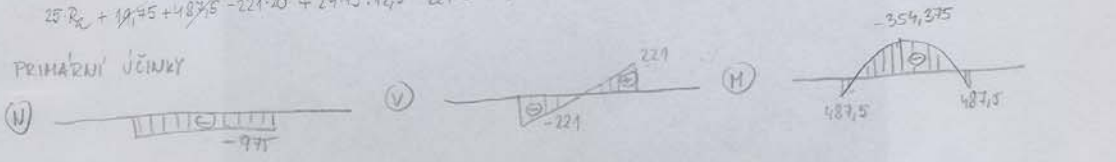
$$\overline{M}_a \cdot \frac{25}{6EJ} + \overline{M}_b \cdot \frac{25}{3EJ} - \frac{246{,}875}{EJ} = 0$$

$$\overline{M}_b \cdot \frac{25}{3EJ} - \overline{M}_b \cdot \frac{25}{12EJ} + \frac{246{,}875}{2EJ} - \frac{246{,}875}{EJ} = 0 \quad \ldots \quad \overline{M}_b = 19{,}75\,kNm$$

$\overline{M}_a = 19{,}75\,kNm$

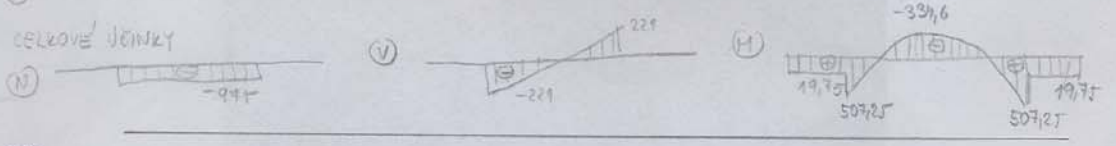
$25 R_a + 19{,}75 + 487{,}5 - 221 \cdot 20 + 29 \cdot 15 \cdot 12{,}5 - 221 \cdot 5 - 487{,}5 - 19{,}75 = 0 \quad \ldots \quad R_a = R_b = 0\,kN$

PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY

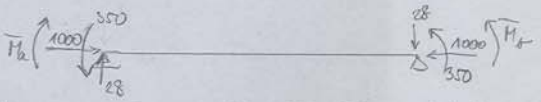
CELKOVÉ ÚČINKY



(27)

$P_H = 1000\,kN$

$2\times SN$



$\frac{0{,}7}{25} \cdot 1000 = P_V = 28\,kN$

$$\overline{M}_a \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{6EJ} + \overline{M}_b \cdot \frac{25}{6EJ} = 0 \quad / \cdot(-\tfrac{1}{2})$$

$$\overline{M}_a \cdot \frac{25}{6EJ} - 350 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{M}_b \cdot \frac{25}{3EJ} = 0$$

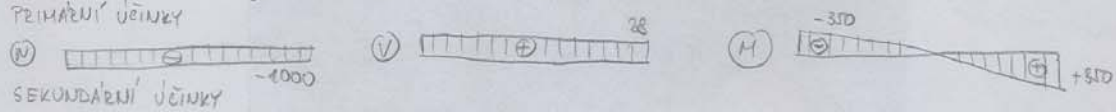
$$-350 \cdot \frac{25}{12EJ} + 350 \cdot \frac{25}{3EJ} + \overline{M}_b \cdot \left(\frac{25}{3EJ} - \frac{25}{12EJ}\right) = 0$$

$25 R_a + 350 - 350 - 350 + 350 + 25 \cdot 28 = 0$

$R_a = -28\,kN \qquad R_b = 28\,kN$

$\overline{M}_b = -350\,kNm \qquad \overline{M}_a = 350\,kNm$

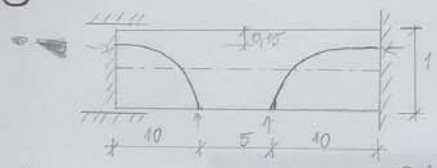
PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

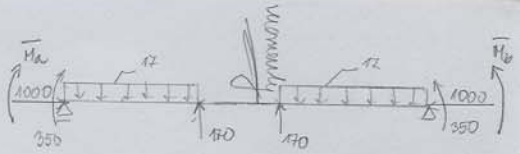
(24)



SN 2.X

$X_1 = \overline{H}_a \quad X_2 = \overline{H}_b$

$P = 1000\,kN$



$tg\alpha = \frac{1,7}{10} = 0,17 \qquad p = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0,85}{20^2} = 17\,kN\,m^{-1}$

$P_V = P \cdot \sin\alpha \doteq 170\,kN$

$\varphi_{ab} = \frac{17 \cdot 10^2}{12EJ}(3 \cdot 25 - 2 \cdot 10) - \frac{170 \cdot 15}{6EJ \cdot 25}(25^2 - 10^2) - \frac{170 \cdot 10}{6EJ \cdot 25}(25^2 - 10^2) = -\frac{14875}{3EJ}$

$\varphi_{ba} = \frac{17 \cdot 10^2}{12EJ}(3 \cdot 25 - 2 \cdot 10) - \frac{170 \cdot 10}{6EJ \cdot 25}(25^2 - 10^2) - \frac{170 \cdot 15}{6EJ \cdot 25}(25^2 - 15^2) = -\frac{14875}{3EJ}$

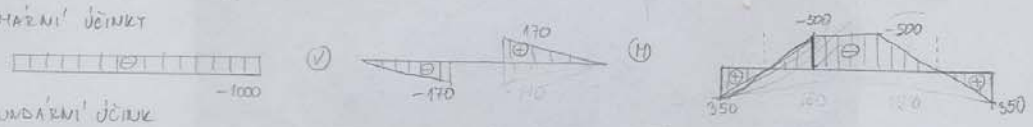
$\overline{H}_a \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{6EJ} - \frac{14875}{3EJ} = 0 \quad / \cdot (-\frac{1}{2})$

$\overline{H}_a \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{3EJ} - \frac{14875}{3EJ} = 0$

$-\frac{875}{3EJ} + \overline{H}_b\left(\frac{25}{3EJ} - \frac{25}{12EJ}\right) = 0 \quad \ldots \quad \overline{H}_b = \frac{140}{3}\,kNm \qquad \overline{H}_a = \frac{140}{3}\,kNm$

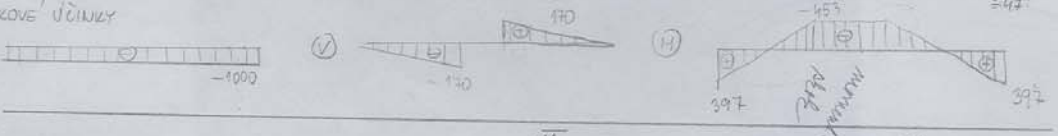
$25 R_a + 350 + \frac{140}{3} - 350 - \frac{140}{3} - 17 \cdot 10 \cdot 20 + 170 \cdot 15 + 17 \cdot 10 \cdot 5 = 0 \quad \ldots \quad R_a = 0 = R_b$

PRIMÁRNÍ ÚČINKY

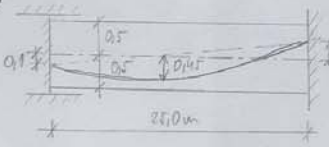


SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



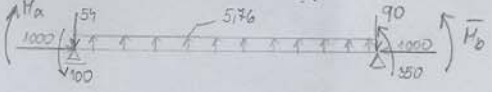
(25)



SN 2.X

$X_1 = \overline{H}_a \quad X_2 = \overline{H}_b$

$P = 1000\,kN$



$y'(0) = \frac{4 \cdot 0,45}{25} + \frac{(-0,35 - 0,1)}{25} = tg\alpha_1 = +0,054 \quad \ldots \quad P_V = P \cdot \sin\alpha \doteq 54\,kN \qquad P_H = P = 1000\,kN$

$y'(25) = -\frac{8 \cdot 0,45}{25} + \frac{4 \cdot 0,45}{25} - \frac{0,45}{25} = -0,09 \quad \ldots \quad P_V \doteq 90\,kN \qquad P_H = P$

$p = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0,45}{25^2} = 5,76\,kNm^{-1}$

$\varphi_{ab} = \varphi_{ba} = \frac{1}{24}\frac{q l^3}{EJ} = -\frac{1}{24} \cdot \frac{5,76 \cdot 25^3}{EJ} = -\frac{3750}{EJ}$

$\overline{H}_a \cdot \frac{25}{3EJ} - \frac{100 \cdot 25}{3EJ} + 350 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{6EJ} - \frac{3750}{EJ} = 0 \quad / \cdot (-\frac{1}{2})$

$\overline{H}_a \cdot \frac{25}{6EJ} - 100 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{3EJ} - \frac{3750}{EJ} = 0$

$\frac{312,5}{EJ} + \overline{H}_b\left(\frac{25}{3EJ} - \frac{25}{12EJ}\right) = 0 \quad \ldots \quad \overline{H}_b = -50\,kNm \qquad \overline{H}_a = 400\,kNm$

$25 R_a - 100 + 50 - 54 \cdot 25 + \frac{1}{2} \cdot 5,76 \cdot 25^2 - 350 + 400 = 0$

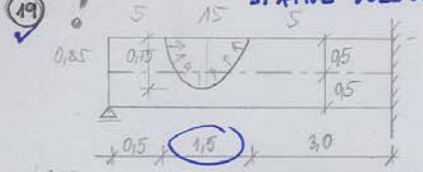
$R_a = -R_b = -12\,kN$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

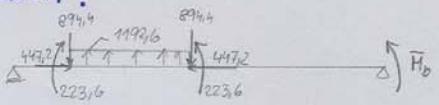
(19) ! ŠPATNĚ VOLENÉ VSTUPNÍ HODNOTY !



SN 1x

$x = \overline{H}_b$

$P = 1000\ kN$



$tg\,\varphi = \frac{1,5}{0,75} = 2$

$P_V = P \cdot \sin\alpha = 894,4\ kN$

$P_H = P \cdot \cos\alpha = 447,2\ kN$

$q = \frac{8 \cdot P_H \cdot f}{l^2} = \frac{8 \cdot 447,2 \cdot 0,75}{1,5^2} = 1192,6\ kNm^{-1}$

$5 \cdot R_a - 894,4 \cdot 4,5 - 894,4 \cdot 3,0 + 1192,6 \cdot 1,5 \cdot 3,75 = 0$

$R_a = -R_b = -0,075 \doteq 0\ kN$

$\varphi_{ba} = \frac{H \cdot l}{6EJ}\left(1 - 3\frac{a^2}{l^2}\right) - \frac{Ml}{6EJ}\left(1 - 3\frac{a^2}{l^2}\right) + \frac{F \cdot a}{6EJl}\left(l^2 - a^2\right) + \frac{F \cdot a'}{6EJl}\left(l^2 - a'^2\right) -$

$- \frac{q \cdot b \cdot c}{24EJ \cdot l}\left[4d\,(l+b) - c^2\right] = \frac{223,6 \cdot 5,0}{6EJ}\left(1 - 3 \cdot \frac{0,5^2}{5^2}\right) - \frac{223,6\ 5}{6EJ} \cdot$

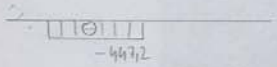
$\left(1 - 3 \cdot \frac{2^2}{5^2}\right) + \frac{894,4 \cdot 0,5}{6EJ \cdot 5}\left(5^2 - 0,5^2\right) + \frac{894,4 \cdot 2}{6EJ \cdot 5}\left(5^2 - 2^2\right) - \frac{1192,6\ 1,5 \cdot 1,25}{24EJ \cdot 5} \cdot$

$\left[4 \cdot 3,75\,(5 + 1,25) - 1,5^2\right] = \frac{-0,0953}{EJ}$

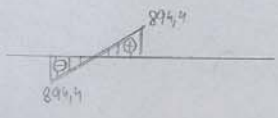
$- \frac{0,0953}{EJ} \doteq 0 + \overline{H}_b \cdot \frac{5}{3EJ} = 0 \quad \ldots \overline{H}_b = 0$

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

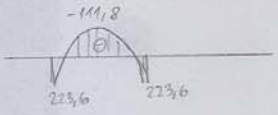
(N)



(V)



(H)



SEKUNDÁRNÍ ÚČINKY

(N)

(V)

(H)

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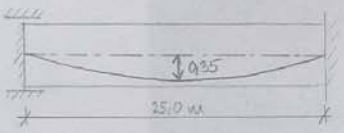
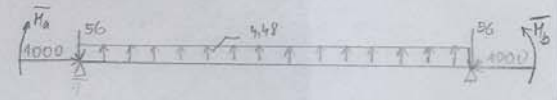
(26)



SN 2x

$x_1 = \overline{H}_a \quad x_2 = \overline{H}_b$

$P = P_H = 1000\ kN$



$y'(0) = y'(25) = \frac{4 \cdot 0,35}{25} = 0,056 = tg\,\alpha \qquad P_V = P \cdot \sin\alpha = 56\ kN$

$q = \frac{8 \cdot P_H \cdot f}{l^2} = \frac{8 \cdot 1000 \cdot 0,35}{25^2} = 4,48\ kNm^{-1}$

$\varphi_{ab} = \varphi_{ba} = \frac{1}{24}\frac{qL^3}{EJ} = -\frac{1}{24} \cdot \frac{4,48 \cdot 25^3}{EJ} = -\frac{8750}{3EJ}$

$\overline{H}_a \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{6EJ} - \frac{8750}{3EJ} = 0 \quad I \cdot (-\frac{1}{2})$

$\overline{H}_a \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{3EJ} - \frac{8750}{3EJ} = 0$

$- \frac{4375}{3EJ} + \overline{H}_b \left(\frac{25}{3} - \frac{25}{12}\right)\frac{1}{EJ} = 0$

$\overline{H}_b = \overline{H}_a = \frac{700}{3} \doteq 233\ kNm$

$25\,R_a + \frac{700}{3} - 56 \cdot 25 + \frac{1}{2} \cdot 4,48 \cdot 25^2 - \frac{700}{3} = 0$

$R_a = -R_b = 0\ kN$

PRIMÁRNÍ ÚČINKY

(N)

(V)

(H)

SEKUNDÁRNÍ ÚČINKY

(N)

(V)

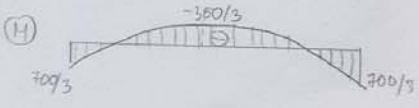
(H)

CELKOVÉ ÚČINKY

(N)

(V)

(H)



(27)

SN 2 x

$X_1 = \overline{H}_a \quad X_2 = \overline{H}_b$

$P_H = 1000\,kN$

$\frac{0,35}{12,5} = \frac{P_V}{1000} \rightarrow P_V = 28\,kN$

$25 \cdot R_a + 350 + 350 = 0 \quad \dots \quad R_a = -R_b = -28\,kN$

$\overline{H}_a \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{6EJ} = 0 \quad / \cdot (-\frac{1}{2})$

$\overline{H}_a \cdot \frac{25}{6EJ} - 350 \cdot \frac{25}{6EJ} + 350 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{3EJ} = 0$

$\frac{2187,5}{EJ} + \overline{H}_b \left(\frac{25}{3EJ} - \frac{25}{12EJ} \right) = 0 \quad \dots \quad \overline{H}_b = -350\,kN$

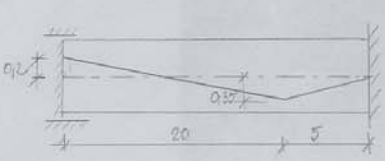
$\overline{H}_a = 350\,kN$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY

(28)



SN 2 x

$X_1 = \overline{H}_a \quad X_2 = \overline{H}_b$

$P_H = 1000\,kN$

$P_V^1 = \frac{0,35}{20} \cdot 1000 = 27,5\,kN \qquad P_V^2 = \frac{0,35}{5} \cdot 1000 = 70\,kN$

$\varphi_{ab} = \frac{F \cdot a}{6EJl} (l^2 - a^2) = \frac{-97,5 \cdot 20}{6EJ \cdot 25} (25^2 - 20^2) = -\frac{2925}{EJ}$

$\varphi_{ba} = \frac{F \cdot b}{6EJl} (l^2 - b^2) = -\frac{97,5 \cdot 5}{6EJ \cdot 25} (25^2 - 5^2) = -\frac{1950}{EJ}$

$\overline{H}_a \cdot \frac{25}{3EJ} + 200 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{6EJ} - \frac{2925}{EJ} = 0 \quad / \cdot (-2)$

$\overline{H}_a \cdot \frac{25}{6EJ} + 200 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{3EJ} - \frac{1950}{EJ} = 0$

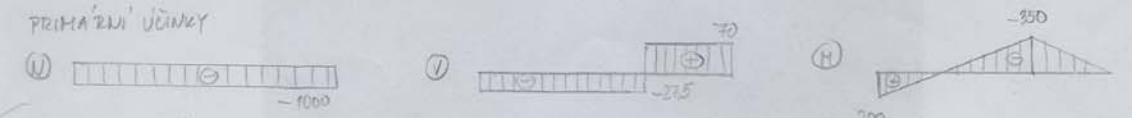
$-\frac{1950}{EJ} + \overline{H}_b \left(\frac{25}{3EJ} - \frac{25}{12EJ} \right) = 0 \quad \dots \quad \overline{H}_b = 312\,kNm$

$\overline{H}_a = -122\,kNm$

$25 \cdot R_a + 200 - 122 - 27,5 \cdot 25 + 97,5 \cdot 5 - 312 = 0$

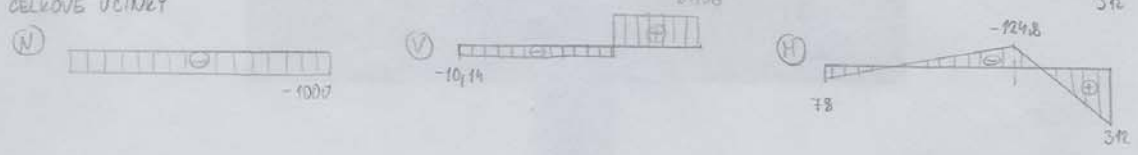
$R_a = -R_b = 17,36\,kN$

PRIMÁRNÍ ÚČINKY

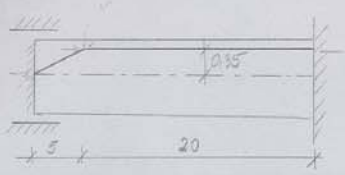


SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



(29)



SN 2x

$X_1 = \bar{M}_a \quad X_2 = \bar{M}_b$

$P_H = 1000\,kN$

$\varphi_{ak} = \frac{Fk}{6EJl}(l^2-b^2) = \frac{70 \cdot 20}{6EJ \cdot 25}(25^2-20^2) = \frac{2100}{EJ}$

$\varphi_{ac} = \frac{Fa}{6EJl}(l^2-a^2) = \frac{70 \cdot 5}{6EJ \cdot 25}(25^2-5^2) = \frac{1400}{EJ}$

$P_V = \frac{0,35}{5} \cdot 1000 = 70\,kN$

$\bar{M}_a \cdot \frac{25}{3EJ} + 350 \cdot \frac{25}{6EJ} + \bar{M}_b \cdot \frac{25}{6EJ} + \frac{2100}{EJ} = 0 \;/\; (-\frac{1}{2})$

$\bar{M}_a \frac{25}{6EJ} + 350 \frac{25}{3EJ} + \bar{M}_b \frac{25}{3EJ} + \frac{1400}{EJ} = 0$

$25 R_a + 70 \cdot 25 - 224 - 70 \cdot 20 - 350 + 406 = 0$

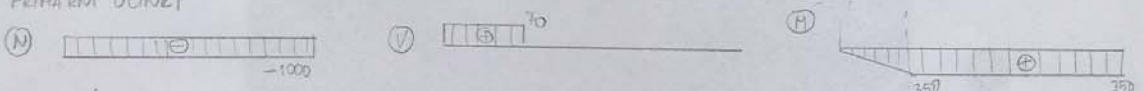
$R_a = -7,28\,kN$

$\frac{2537,5}{EJ} + \bar{M}_b \left(\frac{25}{3EJ} - \frac{25}{12EJ}\right) = 0 \ldots$

$\bar{M}_b = -406\,kNm$

$\bar{M}_a = -224\,kNm$

PRIMÁRNÍ ÚČINKY



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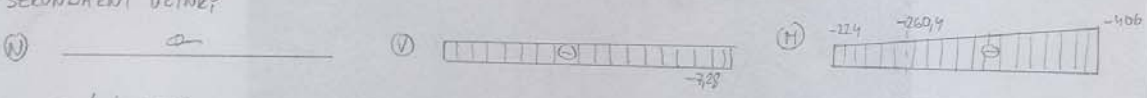
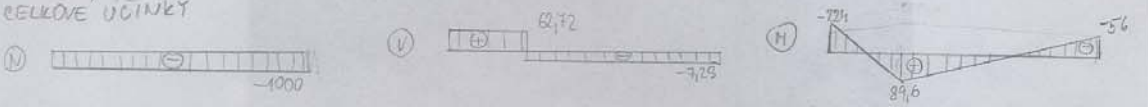


CELKOVÉ ÚČINKY

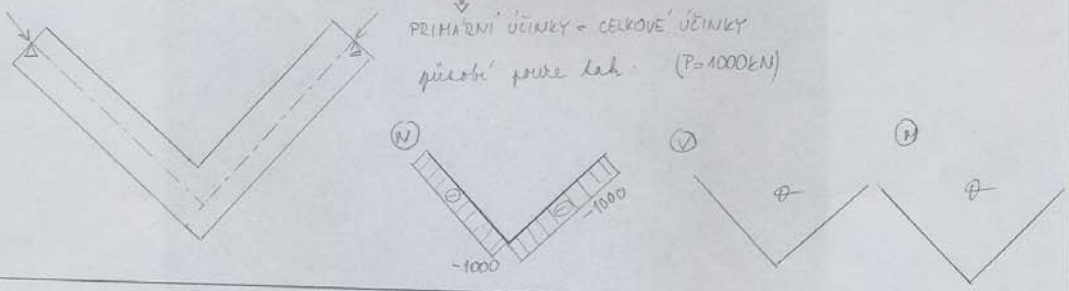


(30)

SN 0x

PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

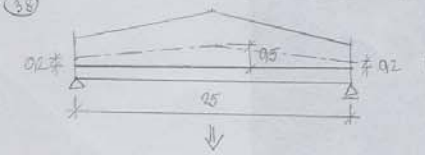
působí pouze tah (P=1000kN)

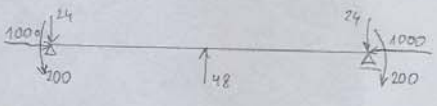


(38)

SN 0x

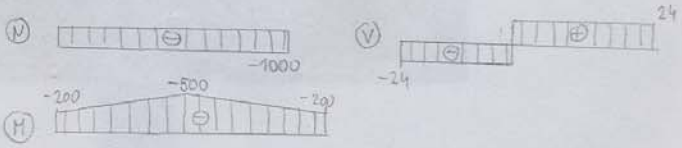
$P_H = 1000\,kN$



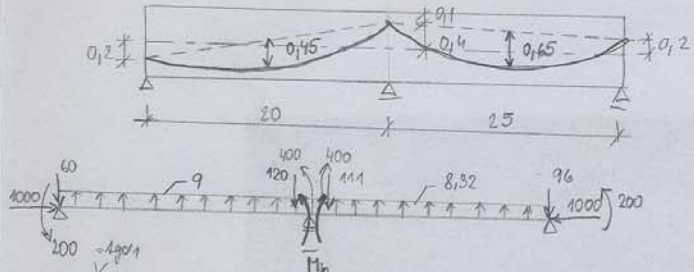


PRIMÁRNÍ ÚČINKY = CELKOVÉ ÚČINKY

$P_V = \frac{0,2}{25} \cdot 2 \cdot 1000 = 24\,kN$



✓ (31)



$P = P_H = 1000\,kN$

SN 1x $\Rightarrow \overline{M}_b = X$

$$y'(x) = -\frac{8fx}{l^2} + \frac{4f}{l} + \frac{e_2 - e_1}{l}$$

$$y_1'(0) = \frac{4 \cdot 0{,}45}{20} + \frac{(-0{,}4-0{,}2)}{20} = 0{,}06 \qquad P_V^1 = P \cdot \sin\alpha \doteq 60\,kN \qquad p = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000\ 0{,}45}{20^2} = 9\,kNm^{-1}$$

$$y_1'(20) = \operatorname{tg}\alpha_2 = -\frac{8 \cdot 0{,}45 \cdot 20}{20^2} + \frac{4 \cdot 0{,}45}{20} + \frac{(-0{,}4-0{,}2)}{20} = -0{,}12 \qquad P_V^2 \doteq 120\,kN$$

$$y_2'(0) = \operatorname{tg}\alpha_3 = \frac{4 \cdot 0{,}65}{25} + \frac{-0{,}2+0{,}2}{25} = 0{,}112 \qquad P_V^3 \doteq 111\,kN \qquad p = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0{,}65}{25^2} = 8{,}32\,kNm^{-1}$$

$$y_2'(25) = \operatorname{tg}\alpha_4 = -\frac{8 \cdot 0{,}65 \cdot 25}{25^2} + \frac{4 \cdot 0{,}65}{25} + \frac{0{,}2}{25} = -0{,}096 \qquad P_V^4 \doteq 96\,kN$$

$$\varphi_{ba} = \frac{1}{24}\frac{p l^3}{EJ} = \frac{1}{24} \cdot \frac{(-9) \cdot 20^3}{EJ} = -\frac{3000}{EJ} \qquad \varphi_{bc} = \frac{1}{24} \cdot \frac{(-8{,}32) \cdot 25^3}{EJ} = -\frac{16250}{3EJ}$$

$$\varphi_{ba} + \varphi_{bc} + M_a \beta_{ab} + \overline{M}_b (\alpha_{ba} + \alpha_{bc}) + M_c \beta_{cb} = 0$$

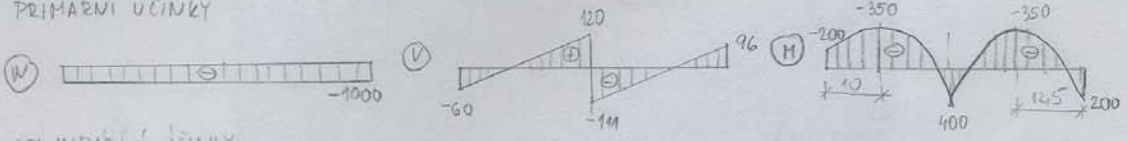
$$-\frac{3000}{EJ} - \frac{16250}{3EJ} - \frac{200 \cdot 20}{6EJ} + \overline{M}_b \left(\frac{20}{3EJ} + \frac{25}{3EJ}\right) + 200 \cdot \frac{25}{6EJ} = 0 \quad \ldots \quad \overline{M}_b = 550\,kNm$$

$$20 R_a - 60 \cdot 20 - 200 + \tfrac{1}{2} \cdot 9 \cdot 20^2 - 550 = 0 \quad \ldots \quad R_a = 17{,}5\,kN\ (\uparrow)$$

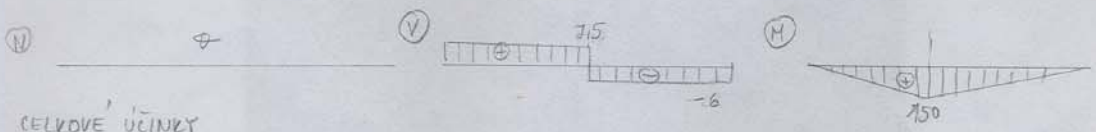
$$25 R_c - 96 \cdot 25 + 200 + \tfrac{1}{2} \cdot 8{,}32 \cdot 25^2 - 550 = 0 \quad \ldots \quad R_c = 96\,kN\ (\uparrow)$$

$\Big\} R_b = 13{,}5\,kN\ (\downarrow)$

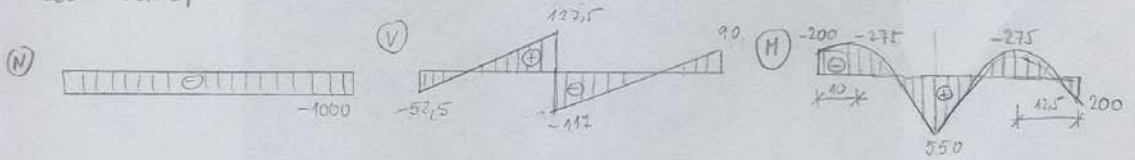
PRIMÁRNÍ ÚČINKY



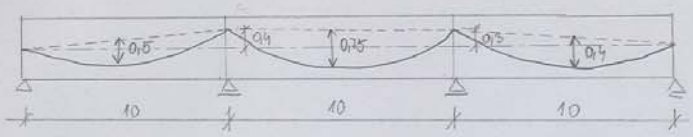
SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY



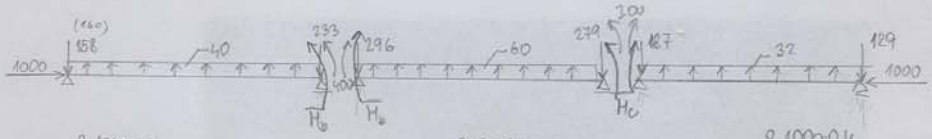
(33)



$P = P_1 = 1000\,kN$

SN 2 x

$x_1 = \bar{H}_b \quad x_2 = \bar{H}_c$



$p_1 = \frac{8 \cdot 1000 \cdot 0{,}5}{10^2} = 40\,kNm^{-1}$ $\quad p_2 = \frac{8 \cdot 1000 \cdot 0{,}75}{10^2} = 60\,kNm^{-1}$ $\quad p_3 = \frac{8 \cdot 1000 \cdot 0{,}4}{10^2} = 32\,kNm^{-1}$

$y_1'(0) = tg\,\alpha_1 = \frac{4 \cdot 0{,}5}{10} - \frac{0{,}4}{10} = 0{,}16$

$P_V^1 = P \cdot \sin\alpha_1 = 158\,kN$

$y_1'(10) = -\frac{8 \cdot 0{,}5 \cdot 10}{10^2} + \frac{2}{10} - \frac{0{,}4}{10} = -0{,}24$

$P_V^2 = 233\,kN$

$y_2'(0) = 0{,}31 \rightarrow P_V^3 = 296\,kN$

$y_2'(10) = -0{,}29 \rightarrow P_V^4 = 279\,kN$

$y_3'(0) = 0{,}19 \rightarrow P_V^5 = 187\,kN$

$y_3'(10) = -0{,}13 \rightarrow P_V^6 = 129\,kN$

$\varphi_{ba} = \frac{1}{24} \cdot \frac{(-40) \cdot 10^3}{EJ} = -\frac{5000}{3EJ} \qquad \varphi_{bc} = \varphi_{cb} = \frac{1}{24} \cdot \frac{(-6) \cdot 10^3}{EJ} = -\frac{250}{EJ} \qquad \varphi_{cd} = \frac{1}{24} \cdot \frac{(-32) \cdot 10^3}{EJ} = -\frac{4000}{3EJ}$

$\varphi_{ba} + \varphi_{bc} + \bar{H}_b^{\,0}\beta_{ba} + \bar{H}_b(\alpha_{ba} + \alpha_{bc}) + \bar{H}_c(\beta_{cb} + \beta_{cd}) + \bar{H}_d^{\,0}\beta_{dc} = 0$

$\varphi_{cb} + \varphi_{cd} + \bar{H}_b(\beta_{ba} + \beta_{bc}) + \bar{H}_c(\alpha_{cb} + \alpha_{cd}) = 0$

$-\frac{5000}{3EJ} - \frac{250}{EJ} + \bar{H}_b \cdot \frac{2 \cdot 10}{3EJ} + \bar{H}_c \cdot \frac{2 \cdot 10}{6EJ} = 0 \quad / \cdot (-\frac{1}{2})$

$-\frac{250}{EJ} - \frac{4000}{3EJ} + \bar{H}_b \cdot \frac{2 \cdot 10}{6EJ} + \bar{H}_c \cdot \frac{2 \cdot 10}{3EJ} = 0$

$-\frac{3750}{3EJ} \cdot (-\frac{1}{2}) - \frac{1}{2} \cdot \frac{20}{6EJ} \cdot \bar{H}_c - \frac{4750}{3EJ} + \bar{H}_c \cdot \frac{20}{3EJ} = 0$

$\bar{H}_c = 12{,}5\,kNm \qquad \bar{H}_b = 22{,}5\,kNm$

$10 R_{ba} = 225 \ldots R_{ba} = 22{,}5\,kN$

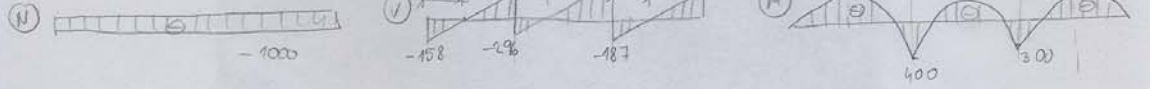
$R_{ba} = -22{,}5\,kN$

$225 + 10 R_{bc} = 125 \ldots R_{bc} = -10{,}0\,kN$

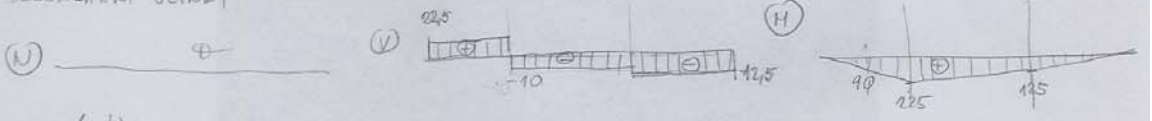
$10 R_{cb} + 125 = 225 \ldots R_{cb} = 10{,}0\,kN$

$10 R_d = 125 \ldots R_d = R_{dc} = 12{,}5\,kN$

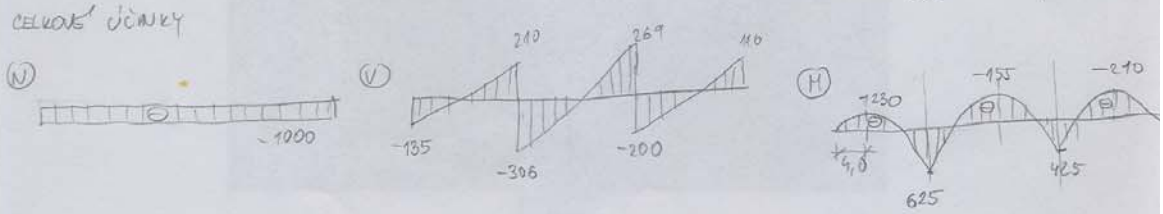
PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY

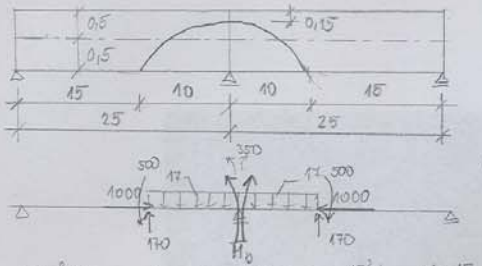


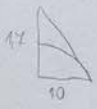
(34)

P = 1000 kN

SN 1x

$x = \overline{H}_b$





$tg\alpha = \frac{17}{10} = 0{,}17$

$P_V = P \sin\alpha \doteq 170\,kN$

$P_H = P \cos\alpha \doteq 1000\,kN$

$\mu = \frac{8 P_H f}{l^2} = \frac{8 \cdot 1000 \cdot 0{,}85}{20^2} = 17\,kNm^{-1}$

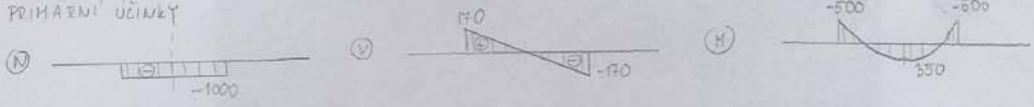
$$y_{ba} = \frac{17\,10^2 \cdot 25}{6EJ}\left(1 - \frac{10}{2 \cdot 25}\right)^2 - \frac{500 \cdot 25}{6EJ}\left(1 - 3\cdot\frac{15^2}{25^2}\right) - \frac{170\,15}{6EJ \cdot 25}\left(25^2 - 15^2\right) = -\frac{2100}{EJ}$$

$$y_{bb} = \frac{17\,10^2 \cdot 25}{6EJ}\left(1 - \frac{10}{2 \cdot 25}\right)^2 - \frac{500 \cdot 25}{6EJ}\left(1 - 3\,\frac{15^2}{25^2}\right) - \frac{170\,15}{6EJ \cdot 25}\left(25^2 - 15^2\right) = -\frac{2100}{EJ}$$

$R_a = R_b = -3{,}93\,kN$

$$\overline{H}_b \cdot \left(\frac{25}{3EJ} + \frac{25}{3EJ}\right) - \frac{2100}{EJ} \cdot 2 = 0 \quad \ldots \quad \overline{H}_b = 252\,kNm$$

PRIMÁRNÍ ÚČINKY



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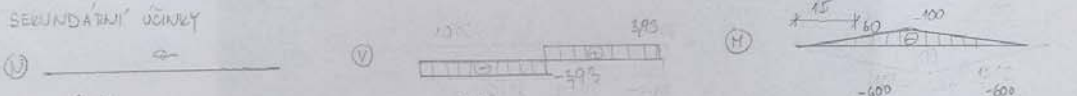
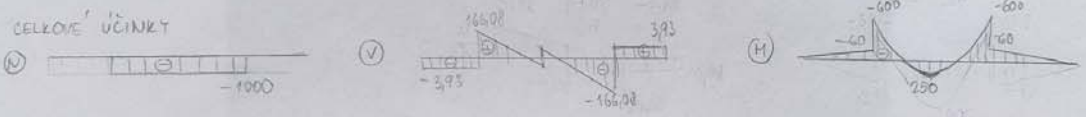


CELKOVÉ ÚČINKY



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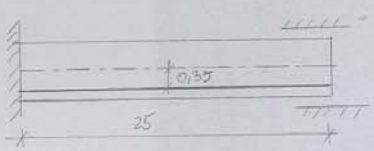
(35)



SN 2x

$x_1 = \overline{H}_a \quad x_2 = \overline{H}_b$

$P_H = 1000\,kN$

$$\overline{H}_a \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{3EJ} + \overline{H}_b \cdot \frac{25}{6EJ} - 350 \cdot \frac{25}{6EJ} = 0 \quad | \cdot \left(-\tfrac{1}{2}\right)$$

$$\overline{H}_a \cdot \frac{25}{6EJ} - 350 \cdot \frac{25}{6EJ} + \overline{H}_b \cdot \frac{25}{3EJ} - 350 \cdot \frac{25}{3EJ} = 0$$

$$-\frac{11875}{EJ} + \overline{H}_b\left(\frac{25}{3EJ} - \frac{25}{12EJ}\right) = 0 \quad \ldots \quad \overline{H}_b = 350\,kNm$$

$\overline{H}_a = 350\,kNm$

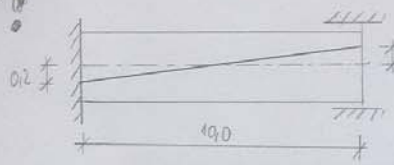
$R_a = R_b = 0\,kN$

PRIMÁRNÍ ÚČINKY

SEKUNDÁRNÍ ÚČINKY

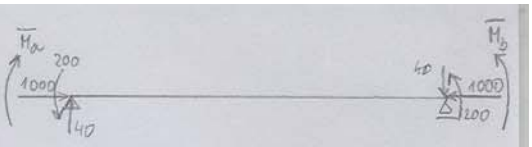
CELKOVÉ ÚČINKY

(36)



SN 2x

$X_1 = \overline{M}_A \quad X_2 = \overline{M}_b$



$P_H = 1000\,kN$

$$\overline{M}_a \frac{10}{3EJ} - 200\frac{10}{3EJ} + \overline{M}_b \frac{10}{6EJ} + 200\cdot\frac{10}{6EJ} = 0 \;/\cdot(-\tfrac{1}{2})$$

$$P_V = \frac{0{,}2}{5}\cdot 1000 = 40\,kN$$

$$10Z_a + 140 - 200 + 40\cdot 10 - 100 + 80 = 0$$

$$Z_a = -R_A = -22\,kN$$

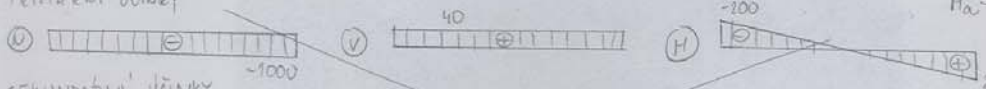
$$\overline{M}_a \frac{10}{6EJ} - 200\cdot\frac{10}{6EJ} + \overline{M}_b \frac{10}{3EJ} + 200\cdot\frac{10}{3EJ} = 0$$

$$\frac{500}{EJ} + \overline{M}_b\left(\frac{10}{3EJ} - \frac{10}{12EJ}\right) = 0 \ldots\ldots \overline{M}_b = -80\,kNm \rightarrow -200\,kNm$$

$$\overline{M}_a = 140\,kNm \rightarrow 200\,kNm$$

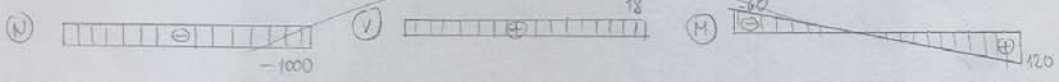
$R_A = -40\,kN$
$R_B = 40\,kN$

PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY

CELKOVÉ ÚČINKY



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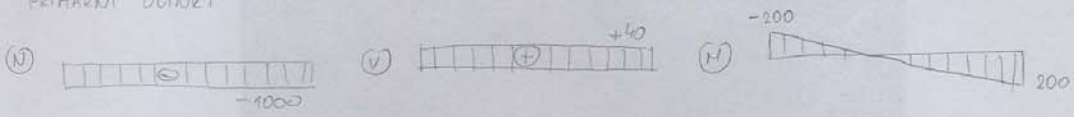

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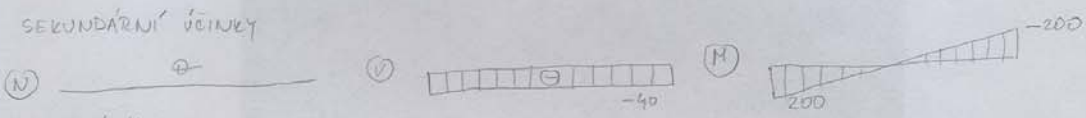
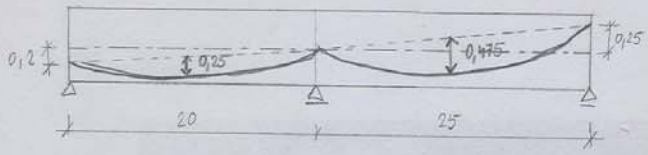


CELKOVÉ ÚČINKY

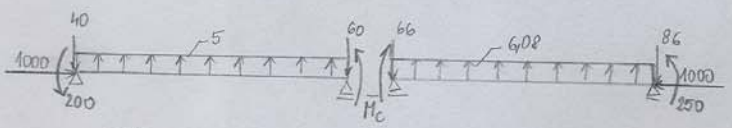
KONKORDANTNÍ KABEL

37



$P = P_H = 1000\,kN$

SV 1x $\Rightarrow$ $X = \overline{H}_b$



$\mu_1 = \frac{8 P_H f_1}{l_1^2} = \frac{8 \cdot 1000 \cdot 0{,}25}{20^2} = 5\,kNm^{-1}$

$\mu_2 = \frac{8 P_H f_2}{l_2^2} = \frac{8 \cdot 1000 \cdot 0{,}475}{25^2} = 6{,}08\,kNm^{-1}$

$y_1'(0) = \frac{4 \cdot 0{,}25}{20} + \frac{(0-0{,}2)}{20} = 0{,}04$ $\quad P_V^1 = P \cdot \sin\alpha_1 = 40\,kN$

$y_1'(20) = \operatorname{tg}\alpha_2 = -\frac{8 \cdot 0{,}25 \cdot 20}{20^2} + \frac{4 \cdot 0{,}25}{20} - \frac{0{,}2}{20} = -0{,}06$ $\quad P_V^2 = 60\,kN$

$y_2'(0) = \operatorname{tg}\alpha_3 = \frac{4 \cdot 0{,}475}{25} - \frac{0{,}25}{25} = 0{,}066$ $\quad P_V^3 = 66\,kN$

$y_2'(25) = \operatorname{tg}\alpha_4 = -\frac{8 \cdot 0{,}475 \cdot 25}{25^2} + \frac{4 \cdot 0{,}475}{25} - \frac{0{,}25}{25} = -0{,}086$ $\quad P_V^4 = 86\,kN$

$\varphi_{ba} = \frac{1}{24} \cdot \frac{(-5) \cdot 20^3}{EJ} = -\frac{5000}{3EJ}$

$\varphi_{bc} = \frac{1}{24} \cdot \frac{(-6{,}08) \cdot 25^3}{EJ} = -\frac{11875}{3EJ}$

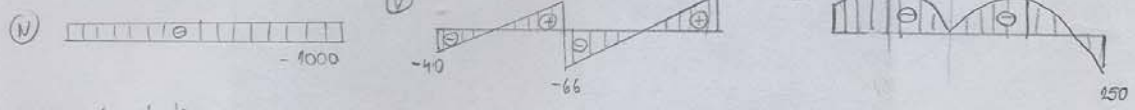
$\varphi_{ba} + \varphi_{bc} + M_a \cdot \beta_{ab} + \overline{M}_b (\alpha_{ba} + \alpha_{bc}) + M_c \beta_{cb} = 0$

$-\frac{5000}{3EJ} - \frac{11875}{3EJ} - 200 \cdot \frac{20}{6EJ} + \overline{M}_b \left(\frac{20}{3EJ} + \frac{25}{3EJ}\right) + 250 \cdot \frac{25}{6EJ} = 0 \ldots\ldots \overline{M}_b = 350\,kNm$

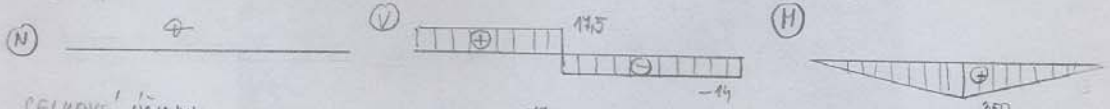
$20 R_a - 200 - 40 \cdot 20 + \frac{1}{2} \cdot 5 \cdot 20^2 - 350 = 0 \ldots\ldots R_a = 17{,}5\,kN\ (\uparrow)$ $\Big\}$ $R_b = 31{,}5\,kN$

$25 R_b + 86 \cdot 25 + 250 + \frac{1}{2} \cdot 6{,}08 \cdot 25^2 - 350 = 0 \ldots\ldots R_b = 14\,kN\ (\uparrow)$

PRIMÁRNÍ ÚČINKY



SEKUNDÁRNÍ ÚČINKY



CELKOVÉ ÚČINKY