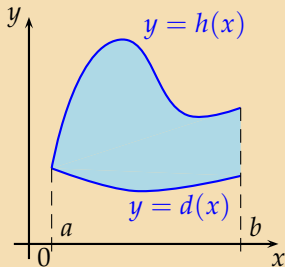


# Obsah rovinného útvaru mezi dvěma křivkami

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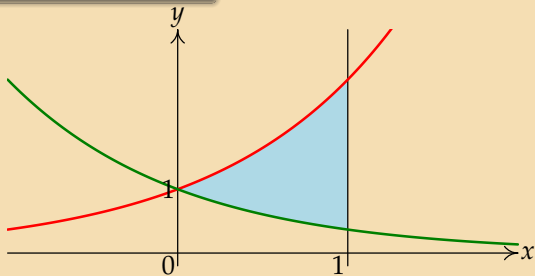
**Obsah rovinné plochy** omezené spojitými funkcemi  $y = d(x)$  a  $y = h(x)$ , které na intervalu  $\langle a, b \rangle$  splňují  $d(x) \leq h(x)$ , a přímkami  $x = a$  a  $x = b$ :



$$S = \int_a^b h(x) - d(x) \, dx$$

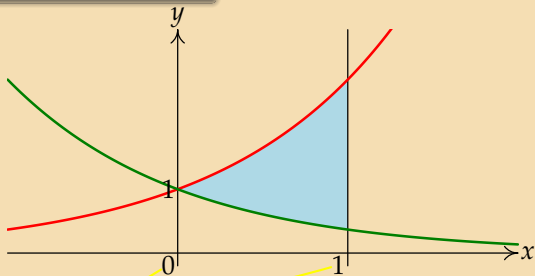
Určete obsah množiny mezi křivkami  $y = e^x$  a  $y = e^{-x}$  pro  $x \in [0, 1]$ .

$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



Zakreslíme křivky.

$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



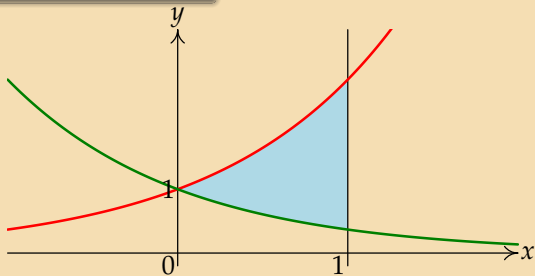
$$S = \int_0^1 e^x - e^{-x} dx$$

Two yellow arrows point from the upper limit '1' and the lower limit '0' of the integral to the x-axis in the graph above. A third yellow arrow points from the  $e^x$  term to the red curve in the graph above. A fourth yellow arrow points from the  $-e^{-x}$  term to the green curve in the graph above.

Vyjádříme obsah plochy jako určitý integrál.

$$h(x) = e^x, d(x) = e^{-x}$$

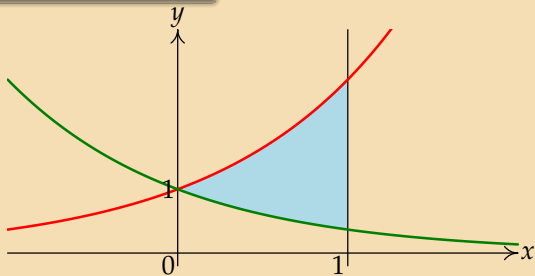
$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



$$S = \int_0^1 e^x - e^{-x} dx = [e^x + e^{-x}]_0^1$$

Vypočteme neurčitý integrál.

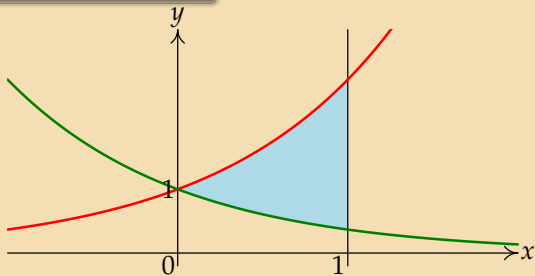
$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



$$S = \int_0^1 e^x - e^{-x} dx = [e^x + e^{-x}]_0^1 = e^1 + e^{-1} - [e^0 + e^0]$$

Vypočítáme určitý integrál pomocí Newtonovy–Leignizovy formule. Dosadíme tedy meze.

$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



$$S = \int_0^1 e^x - e^{-x} dx = [e^x + e^{-x}]_0^1 = e^1 + e^{-1} - [e^0 + e^0] = e + \frac{1}{e} - 2$$

Dopočítáme.

Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .

Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .

$$1 - (x - 1)^2 = -x$$

- První z křivek je parabola, druhá z křivek je přímka  $y = -x$ .
- Křivky se protínají v bodě, jehož  $x$ -ová splňuje rovnici

$$1 - (x - 1)^2 = -x$$

Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .

$$1 - (x - 1)^2 = -x$$

$$1 - (x^2 - 2x + 1) = -x$$

$$1 - x^2 + 2x - 1 = -x$$

$$3x - x^2 = 0$$

$$(3 - x)x = 0$$

Průsečíky křivek jsou body  $[0, 0]$  a  $[3, -3]$ .

Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .

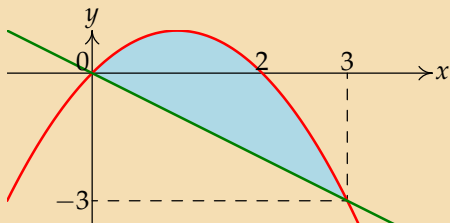
$$1 - (x - 1)^2 = -x$$

$$1 - (x^2 - 2x + 1) = -x$$

$$1 - x^2 + 2x - 1 = -x$$

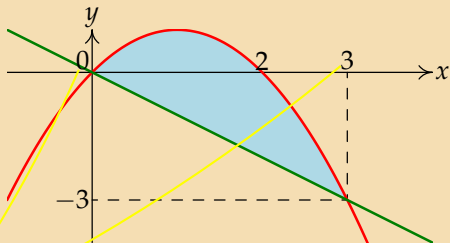
$$3x - x^2 = 0$$

$$(3 - x)x = 0$$



$$y = 1 - (x - 1)^2 = 1 - (x^2 - 2x + 1) = 2x - x^2 = x(2 - x)$$

Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .

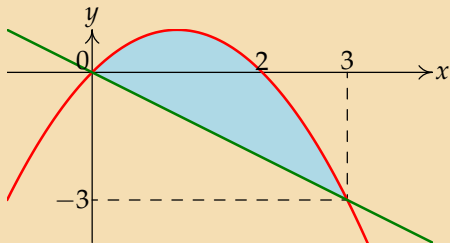


$$S = \int_0^3 1 - (x - 1)^2 - (-x) dx$$

$$h(x) = 1 - (x - 1)^2$$

$$d(x) = -x, \text{ protože } x + y = 0 \iff y = -x$$

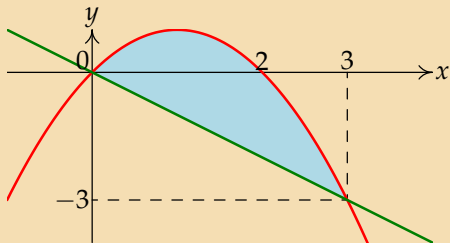
Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .



$$S = \int_0^3 1 - (x - 1)^2 - (-x) dx = \int_0^3 1 - (x^2 - 2x + 1) + x dx$$

Umocníme.

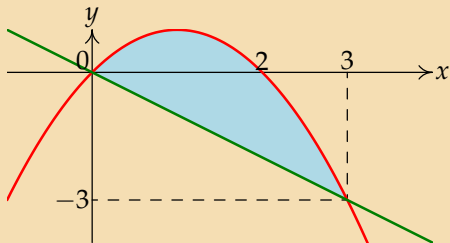
Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) \, dx = \int_0^3 1 - (x^2 - 2x + 1) + x \, dx \\ &= \int_0^3 -x^2 + 3x \, dx \end{aligned}$$

Upravíme integrand.

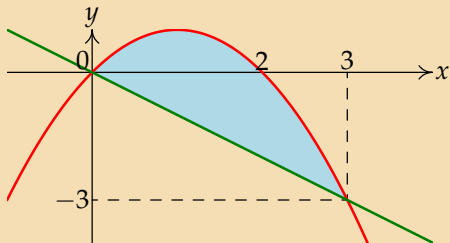
Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) \, dx = \int_0^3 1 - (x^2 - 2x + 1) + x \, dx \\ &= \int_0^3 -x^2 + 3x \, dx = \left[ -\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^3 \end{aligned}$$

$$\int_a^b f(x) \, dx = [F(x)]_a^b = F(b) - F(a)$$

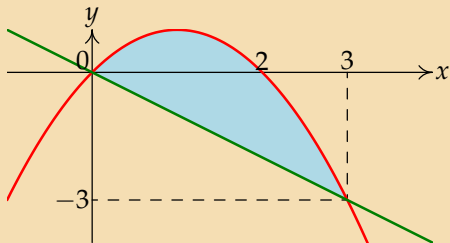
Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) dx = \int_0^3 1 - (x^2 - 2x + 1) + x dx \\ &= \int_0^3 -x^2 + 3x dx = \left[ -\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^3 = \left[ -\frac{3^3}{3} + 3\frac{3^2}{2} \right] - \left[ -\frac{0^3}{3} + 3\frac{0^2}{2} \right] \end{aligned}$$

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

Určete obsah množiny mezi křivkami  $y = 1 - (x - 1)^2$  a  $x + y = 0$ .



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) dx = \int_0^3 1 - (x^2 - 2x + 1) + x dx \\ &= \int_0^3 -x^2 + 3x dx = \left[ -\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^3 = \left[ -\frac{3^3}{3} + 3\frac{3^2}{2} \right] - \left[ -\frac{0^3}{3} + 3\frac{0^2}{2} \right] \\ &= -9 + \frac{27}{2} = \frac{9}{2} \end{aligned}$$

Dopočítáme obsah množiny.

KONEC